AN INTRODUCTION

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Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

CAMBRIDGE SCHOOL AND COLLEGE TEXT BOOKS.

AN INTRODUCTION

TO

PLANE ASTRONOMY

FOR THE USE OF COLLEGES AND SCHOOLS.

BY

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THIRD EDITION.

CAMBRIDGE:
DEIGHTON, BELL, AND CO.
LONDON: G. BELL AND SONS.
1879

PREFACE TO THE FIRST EDITION.

In the present Treatise my object has been to explain as concisely as possible the principles of Plane Astronomy, so as to enable a student, who wishes to enter into either the practical work or the Mathematics of the subject, to do so with a clear knowledge of the connexion between the Astronomical observations and the results to which they lead.

In the few places where I have employed Mathematics to a slight extent, I have done so with a view, not of shewing what the actual calculations are, but of explaining in as simple a way as possible the kind of connexion which exists between the observations and their Mathematical application.

In dividing the subject into chapters, I have followed the arrangement in the work on *Practical and Spherical Astronomy* by my father, the Rev. R. Main. In preparing the Treatise I have examined all the Modern Works on the subject, and in some cases I have quoted from them; the quotations will be found within inverted commas. I have also used, by permission, two or three diagrams from Dean Goodwin's *Course of Mathematics*.

It is hoped that this treatise will answer the purpose of a text-book for that part of the subject which is required in the first three days of the Examination for Mathematical Honours.

Several of my friends have from time to time assisted me with their advice, for which I desire to tender them my most sincere thanks; I am also very greatly indebted to Mr Freeman and Mr Sharpe, both of St John's College, for the care with which they have examined the proof-sheets, and gladly take this opportunity of expressing my obligations to them for their assistance.

P. T. MAIN.

ST JOHN'S COLLEGE,
November 30, 1865.

PREFACE TO THE SECOND EDITION.

IN order to give a wider circulation to this work, the Second Edition has been included in the Cambridge School and College Text Books, at a greatly reduced price. Several fresh articles have been added where they seemed to be required, and the work has been throughout carefully revised.

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ASTRONOMY.

CHAPTER I.

DEFINITIONS AND FIRST PRINCIPLES.

1. *Plane Astronomy; its object.*

ASTRONOMY treats of the motions of the heavenly bodies. All these motions are the results of the Law of Gravitation; and their calculation from that law is the office of Physical Astronomy.

In the present work we treat of the motions of the heavenly bodies as they appear to a spectator on the Earth's surface, of the manner of observing accurately these motions, and of the phenomena to which they give rise. This part of the science is called Plane or Practical Astronomy.

2. *The celestial sphere.*

The first impression which a sight of the heavens on a fine night conveys to the spectator is that he is situated at the centre of a vast hemispherical vault or dome, studded with luminous bodies. He has in fact no means of forming an opinion as to the relative distances from him of the different bodies: he consequently imagines them all to be at the same distance, and to be situated on a spherical surface with himself at the centre: in reality these bodies are at various distances, the nearest of them, the Moon, being about 240,000 miles off, and the greater part of them at distances utterly beyond our powers of measurement.

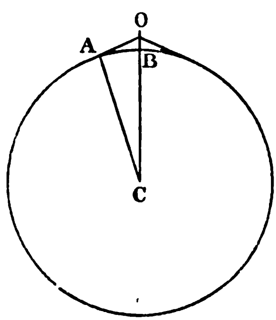
Although this spherical surface is purely imaginary, it will be convenient to retain it for purposes of explanation. We shall therefore frequently indicate the position of a star by the point in which a straight line from the observer's eye to the star meets the surface of a sphere of indefinite radius, and having its centre at the observer's eye. This sphere will be called the *celestial sphere.*

3. *The Earth nearly a sphere; more nearly an oblate spheroid.*

A spectator on land is usually prevented by various irregularities of the surface from forming any conception of the general shape of the visible portion of the Earth. At sea, however, the appearance is that of a plane, bounded in all directions by a circle called the offing, and forming the base on which is supported the visible hemisphere of the heavens.

The earth is, in reality, not a plane, but almost accurately a sphere, the diameter of which is about 7900 miles. The height of the observer above the surface is so small compared with this, that he sees but a very small portion of the Earth's surface, so small indeed that the curvature is almost inappreciable.

To shew this more clearly, let O be the observer's position, C the centre of the Earth: and let OC meet the surface in B. Draw OA a tangent to the Earth.



Then the cosine of the angle ACO is $\frac{CA}{CO}$; which, since

OB is very small compared with CA, is very nearly equal to unity. The angle ACO is, therefore, extremely small; i.e. the curvature of the Earth from B to A is extremely small; and this is true in whatever direction the tangent is drawn from O.

Since the angle ACO is very small, the angle AOC is nearly a right angle, and AO is nearly parallel to the tangent at B.

The angle which AO makes with the tangent at B is called the *dip of the horizon.*

The visible portion of the Earth is bounded by all the points of contact of tangents from O to the Earth. We find therefore that the visible portion of the Earth must appear sensibly a plane; and that the observer's view of the heavens is, neglecting the dip, bounded in all directions by the tangent plane to the Earth at the point immediately beneath him.

We have stated that the Earth is *almost* a sphere: its figure is more exactly an oblate spheroid, i.e. a figure generated by the revolution of an ellipse about its minor axis. The minor axis of the ellipse, or the shortest diameter of the Earth, is about 7900 miles, and is less than the major axis, or the greatest diameter, by about $\frac{1}{300}$ part, or rather more than 26 miles.

This deviation from sphericity is so extremely small, that we shall for general purposes of explanation consider the Earth to be a sphere.

4. *Phenomena confirming the globular form of the Earth.*

Assuming the Earth to be approximately spherical, we should expect to meet with phenomena which are explained by this supposition and are incompatible with the supposition of its being a plane. Such phenomena abound; among the most striking are the following:

(1) The appearance of the offing at sea, as a well-defined circular line.

(2) The observed increase of the dip of the horizon with the height of the observer above the Earth's surface.

(3) The manner of disappearance of a ship as it recedes from sight. At first, the apparent size of each part gets smaller and smaller; this goes on till the ship reaches the offing; after this the hull is observed to disappear bit by bit, from the lowest to the highest part, as it becomes gradually hidden by the sea: in this manner the hull gradually disappears till none of it is seen, while the sails and the masts are still visible; the sails then disappear in the same gradual manner, and leave the top-mast alone visible above the water; this then appears to sink bit by bit, and finally to vanish altogether.

(4) The apparent positions of the Pole-star and of the constellations vary, when seen from different points on the Earth, precisely in the way in which they must appear to vary if these points all lie on the surface of a globe.

(5) The shadow cast by the Earth on the Moon in a lunar eclipse is always bounded by a portion of a circle.

5. *Definitions.*

Since the resultant attraction of a sphere on an external particle is directed to the centre of the sphere, the direction of gravity at any point on the Earth will be the same as the direction of the Earth's diameter through that point. This direction is indicated—almost accurately (Art. 114)—either by a plumb-line, which consists of a fine wire or thread supporting a weight, or by the perpendicular to the surface of some fluid, as water, mercury, or ether, at rest.

Def. The points in which the direction of the plumb-line, or the perpendicular to the surface of a fluid at rest, at any place meets the celestial sphere are called respectively the *Zenith* and the *Nadir;* the zenith being the point immediately above the observer, and the nadir immediately below him.

Def. A plane perpendicular to this line at the observer's position is called the *Sensible Horizon:* and the parallel plane through the Earth's centre is called the *Rational Horizon.*

By what has been said, the Sensible Horizon is a tangent to the Earth's surface, and is the plane which bounds the observer's view of the heavens.

DEF. The great circle in which the celestial sphere is met by either the Rational or Sensible Horizon—See Art. 18—is called the *Celestial Horizon*.

6. *The sphere. Definitions.*

Before proceeding farther it will be desirable to state a few definitions and prove a few propositions relating to the geometry of the sphere.

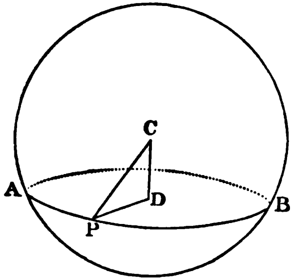
DEF. A sphere is a surface every point of which is equally distant from a point within it called the centre.

It follows at once that every plane section of a sphere through the centre is a circle whose centre and radius are the centre and radius of the sphere.

DEF. Every central section of a sphere is called a great circle.

Hence it follows that all great circles of a sphere are equal; and since any two of them intersect on a diameter of each they bisect each other.

7. *Every plane section of a sphere is a circle.*



For, let APB be the curve in which a plane meets the surface of a sphere, centre C.

Draw CD perpendicular to the plane, and join D with P any point in the curve.

Then since CD is perpendicular to the plane it is perpendicular to the straight line DP in the plane:

$$\therefore CP^2 = CD^2 + PD^2.$$

And CP, the radius of the sphere, is constant: hence PD is constant, and the curve is a circle with centre D and radius equal to PD.

Since DP is less than CP, the radius of a great circle is greater than that of any other section.

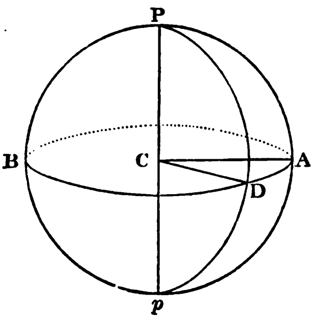
Def. A section of a sphere by a plane not passing through the centre is called a small circle.

8. *Great circles and their poles.*

Def. The poles of a great circle are the points in which a diameter of the sphere perpendicular to the plane of the great circle meets the sphere.

Hence the planes of all great circles which pass through the poles of a great circle are perpendicular to its plane.

Let P, p be the poles of a great circle ADB; C the centre of the sphere; PAp, PDp any great circles through P, p meeting ADB in A and D. Join CA, CD.



Since PC is perpendicular to the plane ADB, AC, DC are both perpendicular to PC; hence $\angle ACD$ is equal to the angle between the planes PAp, PDp. Also PA, pA, PD, pD are quadrants.

Hence a great circle bisects all semicircles joining its poles. And the arc of a great circle intercepted by two planes through its poles subtends at the centre of the sphere an angle equal to the angle between the two planes.

9. *Angular Distance.*

Since the arcs of all great circles are proportional to the angles they subtend at the centre of the sphere, the arc is often used to designate the angle; and the angle is often called the *angular distance* between the extremities of the arc.

10. *The general changes in the appearance of the heavens during a night.*

Unless the contrary be stated the observer will be always supposed to be in north latitude (Art. 12). Suppose, then, an observer in north latitude to watch the appearance presented by the heavens at night during several consecutive hours. If he turns his face towards the south, he will observe on his right hand stars moving obliquely towards and finally disappearing beneath the horizon: on his left he will observe fresh stars appear above the horizon. Each star will appear to move in an arc of a circle, all these arcs being in parallel planes.

If he now turns to the north he will observe that though some of the stars in their downward course disappear below the horizon, others arrive at their lowest point before reaching the horizon, and then move upwards. The motions of these stars will appear also to be in circles or arcs of circles described in parallel planes. Among the stars which do not descend below the horizon, one bright star will appear hardly to move at all, and to be at or about the centre round which all the rest revolve. This is Polaris, or the pole star.

11. *Rotation of the Earth. Definitions.*

Although the stars thus appear to be all in motion, or all with one exception, if their places *among one another* be observed, they will be noticed to remain the same throughout the motion; thus the appearance is as if the whole firmament were rotating bodily about the line drawn from the observer's position to a point near the pole star.

It has been however established beyond doubt that it is the Earth and not the firmament which rotates, and that it rotates with uniform angular velocity: it will be shewn in Chapter IV. how the appearances above described are

explained by supposing the Earth to rotate about an axis which retains a fixed direction in space, and the stars to be at an immense distance.

DEF. The axis about which the Earth rotates is called its *Polar Axis*, and its extremities on the surface of the Earth the *North and South Poles.* The points in which the axis produced meets the celestial sphere are the north and south poles of the heavens.

DEF. A central section of the Earth perpendicular to its axis meets the surface of the Earth in a great circle called the *Terrestrial Equator*, and the celestial sphere in a great circle called the *Celestial Equator.*

DEF. Planes through the Earth's axis meet the surface of the Earth in great circles called *Terrestrial Meridians*, and the celestial sphere in great circles called *Celestial Meridians*, or *Declination Circles.* The Meridian of any place is the Meridian which passes through the zenith of the place.

DEF. The line of intersection of the meridian with the rational horizon is called the *Meridian Line.* The points in which this meets the celestial sphere are called the *north* and *south* points: the north point being that nearest the pole star. If a line be drawn on the horizon perpendicular to this it will meet the celestial sphere in the *east* and *west* points: the east being the point in the neighbourhood of which stars appear to rise above the horizon, and the west the point towards which they set.

12. *Latitude and Longitude.*

The position of a point on the Earth's surface is determined by its Latitude and Longitude, which are defined as follows:

DEF. The *Latitude* of a place on the Earth's surface is the angular distance of its zenith from the equator measured on the meridian; it is therefore the complement of the distance of the zenith from the pole, which is called the Co-latitude. If the place is north of the equator, it is called north latitude; if south, south.

DEF. The *Longitude* of a place is the angle between its meridian and some fixed meridian to which all longi-

tudes are referred. In England the fixed meridian is that of Greenwich, and the longitude of a place is therefore the angle between its meridian and the meridian of Greenwich. A place to the east of Greenwich is said to have east longitude, and a place to the west, west longitude. Both east and west longitudes vary from 0^0 to 180^0.

Def. The small circle in which a plane parallel to the equator meets the Earth's surface passes through places all having the same latitude; it is hence called a *Circle of Latitude.*

13. *Polar-Distance and Hour-Angle.*

Def. The *Polar-Distance* of a star is its angular distance from the pole. The arc of the star's declination-circle intercepted between the star and the equator, which is the complement of the polar distance, is called the *Declination* of the star. The north polar distance of a star is often called its *N. P. D.*; and the south polar distance the *S. P. D.*

Def. The *Hour-Angle* of a star is the angle which its declination-circle makes with the meridian. It receives its name from the fact, that the velocity of the rotation of the Earth being uniform, this angle is proportional to the time elapsed since the star crossed the meridian.

The Polar-Distance and Hour-Angle of a star being known, its place with respect to the observer's meridian is known.

14. *Zenith-Distance and Azimuth.*

There is another method of defining the position of a star with respect to the meridian. It is by the zenith-distance—often called the *Z. D.*—of the star, and its azimuth; which we proceed to define.

Def. The *Zenith-Distance* of a star is its angular distance from the zenith.

Great circles through the zenith are called *Vertical Circles.* The arc of a vertical circle intercepted between the star and the horizon is the complement of the zenith-distance, and is called the *Altitude* of the star.

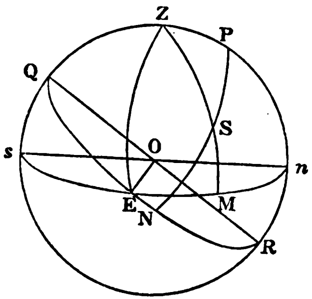
Def. The *Azimuth* of a star is the angle which the vertical circle through the star makes with the meridian.

When the azimuth is measured from the portion of the meridian which passes through the north pole, it is called north azimuth; in the other case, south azimuth.

DEF. The vertical circle which is perpendicular to the meridian of the place is called the *Prime Vertical.*

The prime vertical intersects the celestial horizon in the east and west points.

15. *Illustration of definitions.*



In the figure, Z is the zenith, P the pole, so that PZ is the co-latitude; sEn the horizon, QER the equator; S a star, ZSM a vertical circle, and PSN a declination circle, through S. Pn is the complement of PZ the co-latitude. Thus the altitude of the pole is equal to the latitude of the place.

The great circle $sZPn$ is the meridian, meeting the horizon in s and n the south and north points. PS is the polar distance, SN the declination, $\angle SPZ$ the hour-angle, SZ the zenith-distance, SM the altitude, and $\angle nZS$ the north azimuth.

Since P is the pole of the equator, the meridian $sZPn$ is perpendicular to the equator. Similarly, since Z is the pole of the horizon, the meridian is perpendicular to the horizon; the meridian is therefore perpendicular to OE, the line of intersection of the equator and horizon. Hence the great circle ZE is perpendicular to the meridian, and is the prime vertical.

16. *Sidereal Day.*

By the rotation of the Earth the stars are brought in succession across the meridian of any place: the passage of a star across the meridian is called its *transit;* and the star is then said to transit the meridian.

A star situated in the celestial equator appears in consequence of the diurnal rotation to describe a great circle about the observer's position. The interval between successive transits of a star across the meridian is called a *Sidereal Day;* it is divided into 24 sidereal hours. A star in the equator, therefore, describes about the observer's position angles at the rate of 360^0 in 24 sidereal hours, or 15^0 in 1 sidereal hour.

17. *Circumpolar stars.*

Since all the stars appear to describe circles about *OP* in consequence of the rotation of the Earth, any plane through *OP* bisects all the apparent diurnal circles of the stars; for it passes through the centres of all such circles.

Def. A star, the whole of whose diurnal circle is described above the horizon, is called a *circumpolar star.*

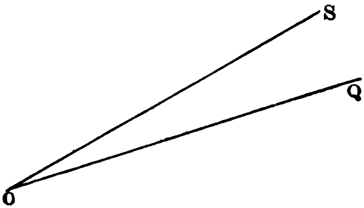
Since any plane through *OP* bisects this circle, the portions of the diurnal circle of a circumpolar star described on each side of the meridian of any place are equal.

18. *Points on the celestial sphere indicate directions.*

A point on the celestial sphere, which has a fixed position among the stars, indicates a fixed *direction* in space; so that a line drawn from the observer's eye to the point will be in the same direction wherever the observer may be. For, let *OS* be a line drawn from the observer to any star, and suppose the observer to change his position by moving to any other point on the Earth's surface: the direction of a line drawn from his new position to the star is the same as before; for the stars are so distant that a diameter of the Earth subtends no appreciable angle at any of them.

Again, it will be explained farther on that the Earth rotates about its axis and describes an orbit round the Sun. But the fixed stars are too far off for even a diameter of the Earth's orbit to subtend any appreciable angle. The direction of *OS* will therefore be unchanged by the Earth's

motion. Hence a line from the observer's eye to any star moves so as to be always parallel to itself.



Again, let OQ be a line drawn from O, the observer's position, in a given direction; and as the observer moves in consequence of the Earth's motion, let OQ be always drawn in the same direction in space: then, since the direction of OS is invariable, the angle SOQ is constant. Hence, if any line OQ moves parallel to itself, it intersects the celestial sphere in a point fixed with respect to the fixed stars.

As an instance of this we may mention the Earth's axis, which is carried by the Earth parallel to itself as the Earth moves round the Sun. The north and south poles of the heavens, being the points in which the Earth's axis meets the celestial sphere, are fixed points among the stars: and the plane of the terrestrial equator which is carried by the Earth in space parallel to itself, meets the celestial sphere in a great circle fixed with respect to the stars. We shall see farther on that the Earth's axis is not *absolutely* invariable in direction, although its change of direction is so slow that the accumulated effect in a whole year is very minute. This change of direction is indicated by a change in the positions of the stars with respect to the poles and equator.

19. *Fixed point.*

By a fixed point on the celestial sphere is to be understood a point which retains an invariable position with respect to the fixed stars; thus, the line drawn from the observer's eye to a fixed point on the celestial sphere has a fixed direction in space, that is, moves parallel to itself.

20. *The actual motions in space are deduced from the observed apparent motions.*

Since the position of an object on the celestial sphere indicates its direction in space as seen from the point of view of the observer, this direction being fixed or variable according as the position of the object with reference to the fixed stars is fixed or variable, it follows that, by noting from time to time the position on the celestial sphere of any heavenly body with reference to the fixed stars, we obtain the changes of direction, if any, of the line drawn from the observer's eye to the object. Hence, the apparent path of a heavenly body among the stars indicates the *angular motion* of the body relatively to the observer; this angular motion may be due partly to the motion of the Earth in space, partly to that of the body. If the body itself were fixed in space there might still be an apparent motion of the body among the stars on the celestial sphere due to the actual motion of the Earth. Again, the body may be moving in space as well as the Earth, while the line joining them moves parallel to itself; the body would then retain a fixed position on the celestial sphere.

We see, then, that the actual motions of the heavenly bodies in space are not given at once by observation of their apparent motions in the heavens; still, these apparent motions are dependent on the actual motions, so that from observations of them at different times and places, the real motions can be inferred; this is in fact the chief object of Practical Astronomy.

21. *Fixed stars. Constellations.*

We have stated that the stars, though appearing to rotate in a body about the axis of the heavens, do not appear to move *inter se*. This is perfectly true of the vast bulk of the heavenly bodies; so that if the position of one of them be noted on any night with respect to certain others, it will be found on all subsequent nights to occupy precisely the same position among them. The stars to which this applies are called, to distinguish them from the few exceptions to the rule, *fixed stars*.

The most conspicuous among the stars have been

from remote antiquity divided into groups, called *Constellations.* All the stars which come within the part of the heavens occupied by a constellation are called by the name of the constellation; the individual star being designated by a letter of the Greek alphabet or a number; e.g. *α Lyræ, α Centauri,* 61 *Cygni.* Some of the most conspicuous stars have proper names besides: as *α Canis Majoris,* which is called Sirius, or the Dog-Star: *α Ursæ Minoris,* called also Polaris, or the Pole-Star.

22. *Magnitude.*

The stars are likewise classified according to their brilliancy. The astronomical term for the brilliancy of a star is *magnitude.* The stars visible with the naked eye are divided into seven classes, according to their magnitude: the most brilliant being those of the first magnitude.

The brightest stars appear to the naked eye to have the largest discs: but this is an optical illusion. On applying telescopes of high magnifying powers, the illusion vanishes, and the stars are seen as points, there being no visible disc at all.

23. *Moon's motion among the stars.*

Some of the heavenly bodies, if watched night after night, will soon be seen to have changed their positions among the stars. Such are the Moon and Planets.

The Moon's motion is so rapid as to be noticeable in a few hours. If its path among the stars were laid down on a celestial globe, it would be found to be a great circle of the celestial sphere. The direction of its motion among the stars is from west to east, or contrary to the direction of the apparent diurnal motion. Motions from west to east are called *direct* motions; those from east to west being called *retrograde.* Thus the Moon's motion among the stars is direct.

The real motion of the Moon's centre in space is approximately in an ellipse of small excentricity with the centre of gravity of the Earth and Moon—a point somewhere within the Earth—in one focus. The mean distance of the Moon is about 60 radii of the Earth, or about 240,000 miles. The angle subtended at the observer's eye by its diameter, called the *apparent* diameter, varies from about

29′ to 33′. Its sidereal period, i.e. the interval between successive arrivals at the same position among the stars, is about $27\frac{1}{3}$ days, and is exactly equal to the time in which it rotates on its axis. We shall reserve further discussion of the Moon for a future chapter.

24. *Sun's motion among the stars.*

The Sun by the brightness of its light quite overpowers the comparatively feeble light of the stars: we cannot therefore observe with the naked eye the position of the Sun with respect to the fixed stars. The general fact, however, that it has a *direct* motion among the stars, i.e. a motion from west to east, can easily be detected by observing stars which rise a little before, or set a little after the Sun. Suppose, for instance, a star be noticed to set a little after the Sun: it is then to the east of the Sun. In a few nights it ceases to be visible, being overpowered by the Sun's rays: soon afterwards it is observed to rise in the east a little before the Sun, and the interval of time by which it precedes the Sun increases daily. The Sun has thus been seen to advance from a position to the west of a star to a position to the east of it: its general motion is therefore direct. Another indication of the Sun's motion among the fixed stars, is the variation of its meridian altitude, which is greatest in summer, and least in winter; the N.P.D. of the Sun is therefore greatest in winter and least in summer.

25. *Ecliptic. Equinoxes. First point of Aries.*

In Chapter III. it will be shewn how by observation of a body at the instant of its crossing the meridian its position among the stars becomes known. If the Sun be so observed from day to day, the successive positions of its centre among the stars are known, and its path among them may be traced. It is found to be a great circle of the celestial sphere.

This great circle is called the *Ecliptic;* the angle which it makes with the equator is called the *obliquity*, and the points in which it intersects the celestial equator are called respectively the *Vernal* and *Autumnal Equinoxes*, the Vernal equinox being the position of the Sun's centre when crossing from the south to the north side of the equator:

the Vernal equinox is also called the *First Point of Aries;* its motion among the stars is so extremely slow, being a result of the change of direction of the Earth's axis, that we shall in general consider it to be a fixed point.

DEF. The angle which a declination-circle through a star makes with that through the first point of Aries is called the *right ascension*, or *R.A.*, of the star.

The apparent motion of the Sun in the heavens is due to the real motion of the Earth in space about the Sun. The fixed stars are at such an immense distance that their positions among one another as seen from the Earth are unaffected by the Earth's change of position in its orbit round the Sun: thus the Sun being seen in different directions and the stars in the same direction, the Sun appears to move among them. For a further explanation of this the reader is referred to Chapter IV.

The orbit of the Earth about the Sun is an ellipse of small excentricity; the mean distance of the Earth from the Sun is about 92 millions of miles.

26. *The planets. Kepler's laws.*

Besides the Sun and Moon there are other bodies called Planets, which are sufficiently near to us to have distinctly visible discs when seen through a telescope of sufficient magnifying power; their motions are not nearly of so simple a nature when viewed from the Earth, as those of the Sun and Moon. Their actual motions in space are, approximately, in ellipses of small excentricity about the Sun; viewed from the Sun their motions would appear equally simple with the motion of the Moon as viewed from the Earth; their apparent motions when viewed from the Earth are all nearly in the plane of the ecliptic, and are sometimes direct, sometimes retrograde. In Chapter VIII. we shall explain these apparent motions, accounting for them by the combined effects of the motion of the Earth and of the Planets round the Sun.

The laws of the motions of the Earth and Planets round the Sun were first discovered by Kepler; they are called Kepler's laws, and are:—

(1) The radii vectores describe areas in one plane proportional to the times of describing them.

(2) The orbits are ellipses with the Sun in one focus.

(3) The squares of the periodic times vary as the cubes of the mean distances from the Sun.

27. *Inferences from Kepler's laws.*

It was afterwards shewn by Newton that it is a necessary consequence of the first law, that the forces under which the planets move tend to the centre of the Sun; of the second law, that these forces vary inversely as the square of the distance; and of the third, that the acceleration with which a planet would be urged towards the Sun at a unit of distance is the same for all the planets, and hence that the masses of the planets are extremely small compared with the mass of the Sun.

28. *Kepler's laws only approximations.*

Kepler's laws, though representing the motions very closely indeed, are only approximations; they would be accurately true if the masses of the planets were infinitesimal. When the masses of the planets, and the effects which they produce, by attracting each other and the Sun according to Newton's law of gravitation, are taken into account, it is found that the minute deviations from Kepler's laws are accurately accounted for.

29. *Satellites.*

Some of the planets have satellites, that is, smaller bodies describing orbits about them, as is the case with the Moon about the Earth: the motions of these satellites are found to be strictly in accordance with the law of gravitation.

30. *The Solar system.*

The Sun, planets, and satellites, including the Earth and Moon, constitute the Solar system. In addition to the bodies enumerated, there are bodies of extremely minute masses which describe parabolas or very long ellipses about the Sun, and are visible when they arrive at the part of their orbit which is near the Sun. These are called Comets.

They really constitute a part of the Solar system; but as they are only occasional visitors, and appear to have no influence on the other members of the system, we shall not discuss them here.

CHAPTER II.

INSTRUMENTS.

31. *Astronomical Instruments; their object.*

The object of Astronomical Instruments is to determine with precision the position of a heavenly body at a given time, or the exact time at which it crosses a given plane.

We propose to describe here the instruments commonly used in a fixed observatory: and as, for astronomical purposes, a means of determining the exact time is indispensable, we shall say a few words on the astronomical clock.

32. *Astronomical Clock. Error. Rate.*

It is essential that a clock to be used for astronomical purposes should go very nearly uniformly. The workmanship must therefore be of the first order, and there must be a contrivance for compensating the effect of changes of temperature: this is effected by a *compensating pendulum,* the most usual form of which is the mercurial pendulum. It consists of a rod to which is attached a cylindrical glass vessel containing mercury: the amount of mercury to be employed is so determined that an increase of temperature by expanding and therefore raising the mercury exactly counteracts the effect produced by the expansion of the rod.

The clock is usually so adjusted as to keep sidereal time: i.e. so that 24 of its hours may elapse between successive transits of the first point of Aries (Art. 25) over the meridian.

The clock should indicate $0^h, 0^m, 0^s$ at the instant of the passage of the first point of Aries.

The interval of sidereal time by which the clock is fast, or the time which it indicates when the first point of Aries transits the meridian, is called the *error* of the clock. If the clock is slow, the error is negative.

The increase of the error in 24 hours is called the *rate* of the clock. If the error diminishes, the rate is negative. By what has been said, it is essential that the rate of the clock should be nearly constant.

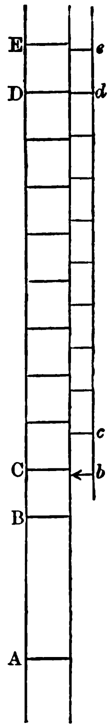
33. *Relation between R.A. of a star and its sidereal time of transit.*

A meridian through any star or any fixed point on the celestial sphere (Art. 19) coincides with the meridian of a place at intervals of 24 hours of sidereal time. The meridian through the first point of Aries, therefore, separates from the meridian of the place at the rate of 360^0 in 24 hours, i.e. of 15^0 per hour: hence the meridian through any star at the instant of its transit will, if the clock has neither error nor rate and indicates t hours, make an angle of $15t^0$ with the meridian through the first point of Aries; i.e. the R.A. of the star will be $15t^0$.

34. *The Vernier.*

Before describing the main instruments used in an observatory, we shall explain two subsidiary instruments, the Vernier and the Micrometer, used for the purpose of measuring small linear distances; the part which they play in the exact determination of the positions of celestial objects will be explained farther on.

Let AE be a straight line perpendicular to which are drawn lines marking off equal intervals, say inches, the inches being measured, from A the first division, upwards: and suppose that it is required to estimate the distance from A of a straight line intermediate between any two divisions, as B and C, to within one nth of an inch. For this purpose a scale is used on which are marked straight lines parallel to each other, and at equal distances such that n of its intervals may occupy the same space as $n-1$ of the intervals in AE, i.e. $n-1$ inches. Each interval of the scale



is therefore an inch *minus* an nth of an inch. Suppose now this scale to be applied with its extremity or zero point b at the position of the line whose distance from A is to be measured. Then one of the divisions of the scale between b and the other extremity e will very nearly coincide with a division of AE: let the divisions D and d be supposed to coincide: then, proceeding from D towards A, the next divisions to D and d will be separated by 1 nth of an inch; the next by 2 $n$ths; and so on: if therefore d be the rth division from b, b and B will be r $n$ths of an inch apart. Thus if $AB = a$ inches, the distance required from $A = a + \frac{r}{n}$, in inches.

The scale be is called a Vernier. If the graduations to be measured are on a circle, the Vernier must be a concentric circular arc. In this case, the graduations may indicate degrees or fractions of a degree: thus, suppose the intervals indicate each 20 minutes, and it is required to measure minutes of arc by means of it. The Vernier must by what has been said contain 19 intervals of the circle in 20 of its own.

35. *The Micrometer.*

The Micrometer appears under various forms, according to the purpose to which it is to be applied. The principle however is the same in all: it consists in measuring, by means of the graduations of a circle attached to the head of a screw, the motion which is communicated by the screw to a wire or spider-line.

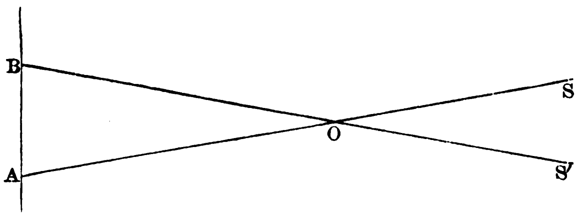
We shall content ourselves with explaining briefly the construction of it as applied to the Transit Instrument. A frame, carrying either one wire, or a couple of intersecting wires, is placed as near the common focus of the object-glass and eye-glass of the telescope as possible without interfering with the transit-wires (Art. 38). If the Micrometer has one wire, it is placed parallel to the transit-wires. The frame containing the micrometer-wire is capa-

ble of motion in a direction perpendicular to the wires, so that the micrometer-wire passes almost in contact with each of the transit-wires. This motion is performed by means of a very fine equable screw, to the head of which is attached a circle which is usually divided into 100 (or 60) equal parts: a pointer fixed to the telescope is so situated that each division of the circle is in turn brought under it by the revolution of the screw.

36. *Value in angle of micrometer-reading.*

By turning the head of the screw until the position of the micrometer-wire coincides in succession with the positions of the transit-wires, the number of revolutions and parts of a revolution of the screw corresponding to the intervals between consecutive wires is easily determined.

Let any two consecutive vertical wires meet the horizontal wire in A and B: and let the telescope be



directed to an equatorial star, and so pointed that the star travels along the horizontal wire, coming successively to A and B. Join A and B with the optical centre O of the object-glass: then AO, BO are the directions of the star when its image is at A and B; and, if produced indefinitely to S and S', they give the directions of the star at those instants. But a star in the equator describes a circle about O in 24 sidereal hours: i.e. it describes 15 seconds of angle in one second of sidereal time. If therefore the interval of time in seconds be observed, which is occupied by the star in its passage between the two wires, we obtain the angle AOB or SOS' in seconds by multiplying by 15.

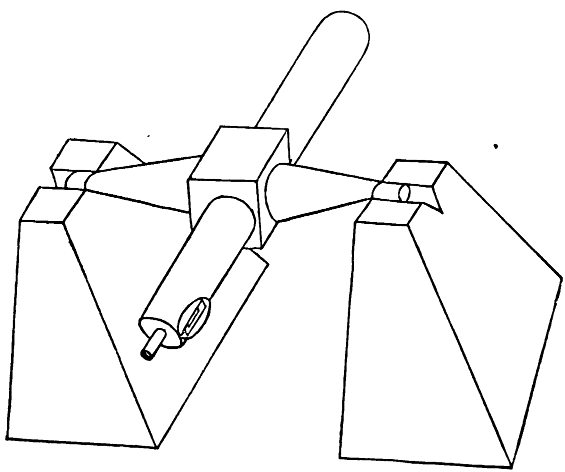
We may thus obtain the value *in angle* of the micrometer-reading from A to B: any other micrometer-reading may hence be reduced to angle by a simple proportion.

37. *The Transit Instrument.*

The object of the Transit Instrument is, in connexion with the sidereal clock, to note the exact time of transit of any star across the observer's meridian.

In fixed observatories, solid foundations are sunk to some distance in the earth, on which are erected stone piers: these piers are constructed to support an astronomical telescope capable of revolving as nearly as possible in the plane of the meridian about a horizontal axis whose extremities rest on the two piers. A telescope so mounted is called a transit instrument. Its axis consists of two cones of metal, of equal dimensions and whose geometrical axes are in the same straight line, firmly fastened to opposite sides of the middle of the tube of the telescope. At the extremity of each of the cones is a cylindrical pivot: the pivots must be of the same diameter and their axes in the same straight line.

The pivots rest on two Y's, as they are called from their



shape, being metallic supports fitting into the piers. Each Y is capable of being moved by a screw—that at the east end vertically, and that at the west horizontally. By means of these screws the axis of the instrument is adjusted so as to be perpendicular to the plane of the meridian; after this has been once done the Y's ought rarely if ever to be moved.

The general arrangement is shewn in the accompanying figure.

38. *Lenses and wires of Transit Instrument.*

The telescope is fitted with a compound object-glass,—combined to correct, as much as possible, chromatism and spherical aberration,—and a Ramsden's or positive eyepiece. At the focus of the object-glass is fixed a frame containing five or seven *vertical wires* (usually made of spider's webs) as nearly as possible equally distant from each other; besides one or sometimes two horizontal wires at about the middle of the field of view.

Besides these there is a micrometer wire, i.e. a wire moveable by a micrometer screw, parallel to and very nearly in the same plane with the vertical wires.

39. *Setting the telescope.*

For the purpose of setting the telescope so as to bring any star, whose N.P.D. is known approximately, into the field of view, there is a small graduated circle rigidly attached to it near the eye-piece, with its plane perpendicular to the axis of rotation of the telescope. Parallel to the plane of this circle and moveable about its centre is attached a spirit-level (Art. 47). The circle is so graduated that when the level is placed with the centre of its bubble at the middle of the tube of the level, a pointer attached to the level and moveable with it indicates on the circle the N.P.D. of a point in the field of view of the telescope.

Hence, if the level be turned about its centre till the pointer indicates the N.P.D. of a star, and the telescope turned so as to bring the bubble into the proper position, the telescope will have the star in its field of view.

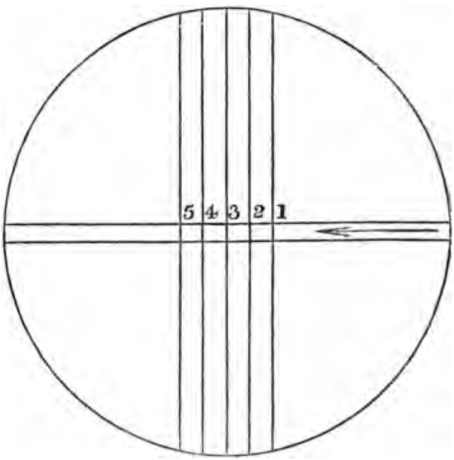
For the purpose of observing at night, one of the pivots of the axis is perforated; by this means the light of a lamp

is admitted and, being reflected at an annular mirror placed in the centre of the axis, illuminates the field of view.

40. *Apparent motion of a star inverted by the telescope.*

The astronomical being an inverting telescope, the image of a star will appear to move across the field of view in the opposite direction to that in which it is seen to move with the naked eye. Thus, if the telescope point to a star in the south, the star will appear to move across the field of view from right to left.

The direction in which a star is seen is the line joining the star's image with the optical centre of the object-glass. If, therefore, one of the wires, say the middle one, were so placed that the plane through it and the optical centre of the object-glass coincided with the meridian, the instant of the star's transit over this wire would be the instant of its transit across the meridian. As this could not be true unless the instrument were in perfect adjustment, which it



is impossible to secure, all that can be done is to place the wire so that this condition may be as nearly as possible satisfied.

41. *Mean of the wires.*

In taking an observation of a star the precise time is noted of its transit over *each* of the wires; and the mean of these times is found. By this means errors of observation at some wires will probably be partially balanced by opposite errors at others, and the residual error will be divided by the number of the wires; on which account the amount of error in the result is diminished considerably.

The time so found will practically very nearly, but probably not accurately, coincide with the time of transit over the middle wire: it will in fact be the time of transit over an imaginary wire very near the middle wire. This imaginary wire is called the *mean of the wires.*

If this wire were in the meridian plane through the optical centre of the object-glass, the result would be the time of transit over the meridian. This is never accurately the case: but the deviation from the meridian can be found, and its effect on the time of any star's transit applied as a correction.

42. *Adjustment of the wires.*

The 'vertical' wires of the telescope are adjusted by directing it to some distant terrestrial object. If one of these wires passes through the image of some well-defined point of this object, and, on turning the telescope about its axis, continues to pass through it, the wire is strictly perpendicular to the axis. If the wire moves away from the image of the point, the frame must be shifted, and the observation repeated until the condition is satisfied. The vertical wires are made as accurately as possible parallel to each other, and the horizontal wires perpendicular to them: so that if one of the vertical wires is accurately adjusted the whole set may be considered to be so.

The 'horizontal' wire, being thus made parallel to the axis of rotation of the telescope, is affected with any error of direction which affects the axis. If the axis is accurately horizontal, the wire is accurately horizontal: if not, the horizontal wire is inclined to the horizon at an angle equal to the error of level (Art. 45) of the axis.

43. *Taking a transit.*

We shall now proceed to explain the process of taking a transit.

The observer, having set the telescope to the proper N.P.D. by means of the attached circle, notices, shortly before the star is expected to cross the meridian, the hours, minutes, and seconds denoted by the clock; he then proceeds to count the seconds as they are ticked by the clock until he sees the star arrive at the first wire; this will generally be in an interval between two successive beats of the clock, so that a fraction of a second has elapsed since the last beat. To estimate this fraction the observer mentally compares the distances of the star from the wire at the consecutive seconds (say for example 14 and 15) immediately before and after its transit over the wire, and taking those distances to be proportional to the intervals in time, writes down his result for the time of transit over the first wire; e.g. 14·7 seconds. He proceeds in the same way to estimate the times of transit over all the wires in succession. By adding these times and dividing by the number of wires, he obtains the time of transit over the mean of the wires, from which, by applying the *corrections*, the time of transit over the meridian is determined.

44. *Three corrections necessary.*

Before considering the above-mentioned corrections, it will be necessary to give the following definitions.

Def. The *line of sight* is the straight line joining the optical centre of the object-glass with the intersection of the horizontal and mean vertical wires.

Def. The *plane of collimation* is the plane through the optical centre of the object-glass perpendicular to the axis of rotation of the telescope.

Def. The *line of collimation* is the straight line joining the optical centre of the object-glass with the point of intersection of the plane of collimation with the horizontal wire.

Three corrections are necessary, due to the impossibility of satisfying with mathematical accuracy the three following conditions:

(1) The perpendicularity of the axis of rotation of the telescope to the *line of sight.*

(2) The horizontality of the common axis of the pivots, —which is the axis of rotation of the telescope.

(3) The coincidence of the plane described by the *line of sight* in a revolution of the instrument, with the meridian.

45. *Errors of adjustment.*

Hence we have the three following *errors of adjustment:*

(1) *The Error of Collimation;* or the inclination of the line of sight to the plane of collimation.

(2) *The Error of Level;* or inclination of the axis to the horizon, or, which is the same thing, the inclination of the plane of collimation to the vertical.

(3) *The Azimuthal Error;* or *Error of Deviation;* being the angle which the plane described by the line of sight makes with the meridian.

In consequence of these errors the actual time of transit of a star over the mean of the wires will be different from its time of transit over the meridian: and corrections will therefore have to be calculated on account of these three causes. In practice the instrument is kept in such good adjustment that the errors are all small: we may therefore practically calculate the correction due to each error separately and add the results. The whole correction so obtained will be near enough to the truth for all practical purposes.

46. *Error of Collimation determined.*

We shall now explain the methods employed for detecting and measuring these several errors.

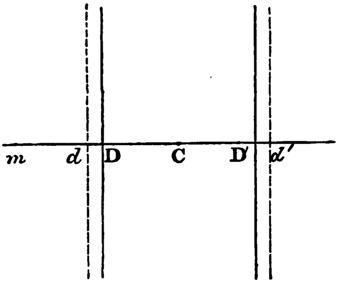
(1) To detect and measure the error of collimation. This is effected in one or other of two ways: either by collimating marks or collimating telescopes.

A collimating mark is a small object placed at a considerable distance from the telescope, and in such a position that the telescope may be directed so as to have it nearly in the middle of its field of view.

Let m be such a mark: and let the telescope be pointed to it so that the horizontal wire may *bisect* it (i.e. pass through it): also let D be the intersection of the horizontal

wire with the middle vertical wire, and *C* with the plane of collimation.

Let now the axis of the telescope be reversed, so that the east pivot points west, and *vice versâ:* then, if the radii of the pivots be accurately equal, the axis of rotation of the telescope is unaltered in direction, and the plane of collimation which is perpendicular to it is also unaltered:



it will therefore meet the horizontal wire at the same distance from *m* on the same side as before, i. e. at *C*. But all points of the instrument on either side of the plane will be brought to the same distance on the other side. Thus *D* will, after the telescope is reversed, appear in the field of view at *D'*; *CD'* being equal to *CD*.

Now, before reversing the telescope, we measure the interval *mD* by moving the micrometer-wire from *m* till it coincides with the middle wire. The same is done after reversal. If the two readings are equal, *D* coincides with *C*, and the middle wire is accurately in the plane of collimation. If they differ, their difference is the micrometer-reading corresponding to *DD'*, or twice *CD*. Half the difference of the two readings will therefore correspond to the interval between the middle wire and the plane of collimation; and, if reduced to angle as in Art. 36, will give the error of collimation.

This supposes observations to be referred to the middle wire. If, as is usual, the mean of the wires be used, let it be represented by the dotted line meeting the horizontal

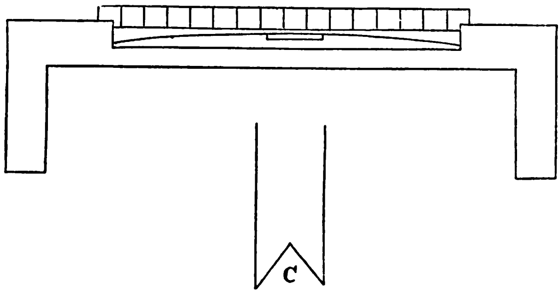
wire in d before, and d' after, reversal; it will be necessary then to find its interval from D and add that interval to CD obtained as above. The interval Dd is easily found by taking the difference of the readings of the micrometer-head, for the middle wire, and for the mean of the wires found as in Art. 41.

The method of *collimating telescopes* will be explained under the head of the Transit Circle.

47. *Spirit-level.*

(2) To detect and measure the error of level.

This is done by means of a spirit-level. The ordinary form of the spirit-level as applied to the Transit Instrument is a glass tube nearly full of ether; the tube is very nearly cylindrical, but bent in the form of an arc of a circle of large radius.



This tube is supported and firmly fastened in a horizontal bed of metal, which leaves its upper surface (which is convex) uncovered. To each extremity of the metal bed is attached a vertical leg; the legs are of equal length, and each terminates in a kind of foot of the form represented by C in the figure. The distance between the legs must be equal to the length of the axis of the instrument, so that the feet of the level may be made to stand on the pivots. Immediately above the bubble is fixed an ivory divided scale which is graduated from the middle point in both directions; the space between successive graduations being a portion of the circular arc into which the tube is

bent, subtending an angle of 1″ at its centre. The tube being not quite full of ether, there will be a small bubble of air, which will rise, in every position of the level, to the highest point of the tube.

A level of this description is called a striding-level. Near one end of the glass tube is attached a small level whose axis is perpendicular to the plane of the striding-level: the object of this additional level is to secure the verticality of the plane of the striding-level.

48. *Angular motion of the level in its own plane given by the mean of the readings of the two ends of the bubble.*

We shall now shew the effect on the position of the bubble of a tilt of the level in its own plane.

Fig. 1.

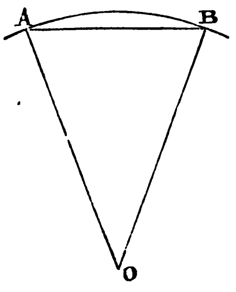
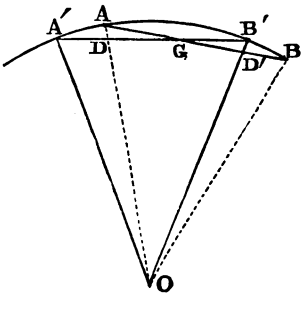


Fig. 2.



Let A, B fig. 1 be the extremities of the bubble, O the centre of the circular arc formed by the tube: suppose the tube to be shifted so that the points A, B of the tube come into the positions indicated in fig. 2, and let $A'B'$ be the position of the bubble after the shift: then if AB meet $A'B'$ in G, the angle AGA' is the angle through which the level has been shifted. But if the bubble occupies the same length in both cases $AB = A'B'$, and therefore the angles BAO, $B'A'O$ are equal: hence the triangles ADG, $A'DO$ have the angles at A and A' equal, and also the angles at D equal: therefore the angle AGA' is equal to

the angle AOA', which is equal to the angle BOB'. Thus the number of seconds indicated by the number of graduations passed over by either end of the bubble is equal to the number of seconds through which the level has been turned.

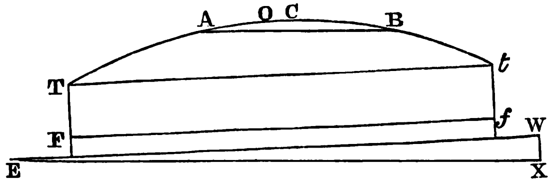
And this angle is clearly equal also to the angle indicated by the number of graduations passed over by the *middle point* of the bubble. Also, if change of temperature by expanding or contracting the ether has altered the size of the bubble, its middle point will not be altered in position thereby, for it will be still the highest point of the tube. Thus the motion of the middle point of the bubble will accurately measure the angular shift of the level, and will give a result unaffected by change of temperature. On this account both ends of the bubble are read, and the reading of the middle point inferred.

49. *Level-error determined.*

Suppose, now, that we wish to determine the level-error of a transit instrument.

The level is made to stand with its two feet on the pivots of the axis, and its plane is made vertical by moving it round to such a position that the extremities of the bubble of the small level are equidistant from the middle point of its tube.

Let F, f be the angular points of the feet of the level,



so that the straight line Ff is parallel to the axis of rotation of the telescope, EW; Tt the axis of the tube of the level, AB the position of the bubble, C its middle point. Suppose EW to be inclined to the horizon, the west end being higher than the east, and let a vertical plane through EW meet the horizontal plane through E in EX.

The graduations are read at A and B, and their difference is taken, which gives twice the distance by which C is west of O the zero of the graduations.

The level is now reversed, so that the foot which was east is placed on the west pivot, and *vice versâ:* F and f have therefore changed places, and the centre of the bubble is still to the west of O, and is therefore at a graduation of the tube between O and T. The readings of the end of the bubble are taken again, and their difference found, which gives twice the distance from O of the centre of the bubble when the level is reversed.

The sum of these two differences being found, gives twice the whole change of position of the centre of the bubble, in consequence of reversing the level. But, by what has been shewn (Art. 48), the number of graduations moved over by the centre of the bubble is equal to the number of seconds of angle through which the level has been shifted. Now, by the reversal, Ff has been brought just as much below a horizontal plane through F as it was above it. The level has therefore been shifted through twice the inclination of Ff to the horizon: i.e. through twice the angle WEX, or twice the level-error.

The result, therefore, obtained by summing the difference of the readings in the two cases is equal to four times the level-error.

50. *Essentials of a good spirit-level.*

It will be observed that it is essential to the success of this process that F and f should be exactly at equal distances from the axis of the telescope, and consequently that the angles of the forks which constitute the feet of the level should be accurately equal. It is not essential that the axis Tt of the level should be accurately parallel to Ff, i.e. that the legs should be of exactly equal length; but it must be in the vertical plane through the axis of rotation of the telescope.

In practice several determinations are made by the above method, and the mean of the whole taken, by which means purely accidental errors will almost disappear; for those which tend to make the result too large may be expected to be on the whole counterbalanced by those

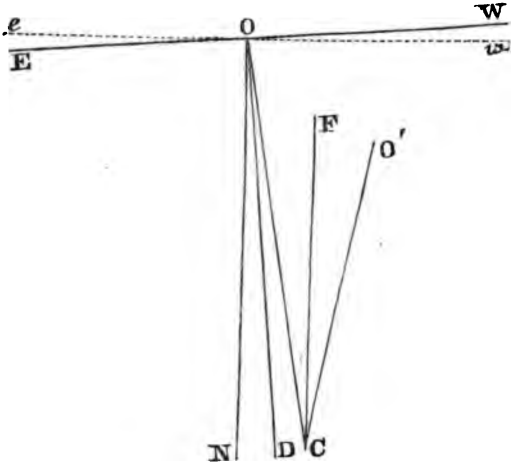
which tend to diminish it, and any residual error will be divided by the number of observations taken.

51. *Bohnenberger's (or collimating) eye-piece.*

Having determined either the error of collimation or the error of level by the preceding or any other method, the remaining error may be determined by a contrivance called the collimating (or Bohnenberger's) eye-piece.

This is an eye-piece of three lenses, fitted into a tube which is perforated to admit the light of a lamp; between the middle lens and the field-lens is placed, at an angle of 45° with the optical axis of the eye-piece, a translucent plane glass reflector.

When an observation is to be taken with this eye-piece, it is substituted for the usual eye-piece, and the telescope is pointed vertically downwards upon a trough of mercury. Near the hole in the tube of the eye-piece is placed a lamp, whose light, incident horizontally on the mirror, is reflected vertically down upon the wires of the telescope. The telescope is brought into such a position that the image of



the 'horizontal' wire coincides with the wire itself. The line of sight is then in a vertical plane through the horizontal wire: hence, since the horizontal wire is parallel

to the axis of rotation (Art. 42), the line of sight and axis of rotation are in the same vertical plane.

The central vertical wire is seen in the field of view illuminated by the lamp: and the reflection from the mercury of the under surface of the wires shews the dark reflected image of the central wire side by side with the wire itself.

Let now EW (in the plane of the paper) be a line parallel to the axis of rotation, through O the optical centre of the object-glass, ON also in the plane of the paper the vertical through O, OC the line of sight meeting the mercury in C; then OC, by what precedes, is in the vertical plane through EW, i.e. in the plane of the paper; and thus the reflected direction $O'C$ is also in this plane.

Let OD be the projection of OC on a plane perpendicular to EW or the plane of collimation; thus OD is in the plane containing OC and EW, which is the plane of the paper.

Draw ew in this plane perpendicular to ON; then ew is the projection of EW on a horizontal plane through O. Thus, $\angle NOD = \angle eOE =$ the error of level; also $\angle COD =$ error of collimation of the middle wire.

Hence $\angle CON =$ error of level + error of collimation.

It has been supposed here that the *west* end of the axis is the higher end, and that the plane of collimation meets the horizontal wire to the *left* of the central wire. In other cases, the $\angle CON$ may be the difference, instead of the sum of the errors. It is easily seen, by inspection of the figure in each case, whether it is the sum or difference.

The frame containing the wires of the transit instrument is moveable in its own plane by a micrometer-head attached to it. This micrometer-head is turned until OC coincides with its reflected direction, i.e. till OC coincides with ON, and the reading taken: in practice the *mean* of the readings is taken for the positions in which the wire and its image touch, first on one side and then on the other.

The number of turns of the micrometer and the reading indicated by the pointer will therefore determine the number of seconds in the $\angle CON$, i.e. in the sum of the errors of collimation and level. Hence, having determined one of these errors by some other means, Bohnenberger's eye-piece enables us to find the other. If the error of collima-

tion be given, it must first be corrected for the interval between the mean of the wires and the middle wire; and if the error of level be given, the error of collimation deduced by this method must be so corrected.

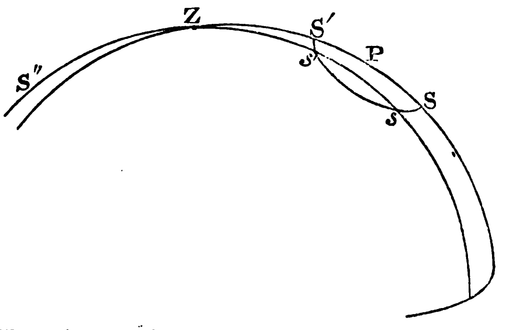
52. *Coincidence of the horizontal wire with its image not necessary in practice.*

By the rotation of the telescope about EW, OC describes a right circular cone with EW as axis. The inclination of OC to the vertical will therefore increase as the telescope moves in either direction; and the inclination of OC to the vertical when OC is in the plane WON is a minimum. The angle OCF will therefore vary *very slightly* for a small motion of the telescope. The distance of the central wire from its image, when the telescope is pointed vertically downwards on the mercury, will therefore not vary perceptibly for a small motion of the telescope about EW. Thus it is unnecessary practically to make the horizontal wire exactly coincide with its image; and if this wire and its image are in the field of view together, the above method gives accurately twice the angle CON.

53. *Azimuthal error, or error of deviation, determined.*

(3) To detect and measure the *azimuthal error*, or *error of deviation.*

This may be done in a variety of ways.



The principle of the various methods may be thus explained. Let Z be the zenith of the place of observation; P the north pole. The meridian ZP bisects the diurnal

paths of all the stars (Art. 17). Any other vertical plane divides all the diurnal circles, with the exception of that of stars in the equator, into unequal parts; for it meets the axis of the heavens in one point only, which is the centre of the celestial sphere and therefore of the diurnal circles of stars in the equator: it cuts the planes of all other diurnal circles in lines not passing through their centres, and divides their circumferences into unequal arcs. The arcs being unequal, the times of describing them are unequal.

Now, supposing the azimuthal error alone to exist, the line of sight of the transit instrument describes a great circle passing through the zenith, which circle does not accurately coincide with the meridian. Let *Zs* be the circle, the deviation being in this case towards the west; then from what has been said, the times of describing the two parts into which the path of any star is divided by the plane *Zs* are unequal; the whole path is described in 24 hours; the two parts are therefore described one in less and the other in more than 12 hours.

If, then, the times of transit of any circumpolar star (i. e. one whose whole path is above the horizon) be observed at consecutive superior and inferior transits, as at s' and s in the figure, the amount by which the interval between the two transits differs from 12 hours will depend (for the same star) on the angle which *Zs*, the plane of collimation of the telescope, makes with the meridian, i. e. on the azimuthal error. By suitable formulæ, given the N.P.D. of the star and the interval between its transits, the azimuthal error may be determined.

54. *Stars near the pole to be preferred.*

It has been remarked that the diurnal circles of stars in the equator are bisected by *Zs*: the method is therefore inapplicable to such stars. And the farther a star is from the equator the more does the interval between the superior and inferior transits differ from 12 hours; and thus any error in the assigned interval will bear a less proportion to this difference, and therefore have less influence on the result, the farther the star is from the equator. Hence it is desirable to observe a star near the pole: the most convenient is Polaris, whose N.P.D. is about $1\frac{1}{2}^{\circ}$, and which has also the advantage of being visible through the telescope in *the day-time.*

55. *Another method.*

It is clear that, if the Transit Clock were accurate, or if we knew its error and rate, we might from *one* transit alone of Polaris determine the azimuthal error. For the Nautical Almanac gives its R.A.; and this, expressed in degrees and fractions of a degree, and divided by 15, gives the time in hours of its superior transit (Art. 33). The preceding method, however, has the advantage of being independent of the clock-error and of the R.A. of the star, or of the position of the first point of Aries.

Another method of determining the azimuthal error is by observing the times of transit of two stars; the difference of their R.A.s gives the difference of the times of transit over *the meridian:* the amount by which this differs from the observed interval is due to the azimuthal error, which may be calculated from it.

56. *Personal equation.*

Besides the instrumental errors there is another affecting all observations of transits, called *Personal Equation.* This error arises from the fact that different observers vary in the degree of rapidity with which they connect the sound of the clock with the place of the star in the field of view at the instant at which the beat of the clock indicating the second is heard. This difference, of course, is very small in general; and in the average of a considerable number of observations it is for the same person pretty constant. When two observations of transit are recorded it is necessary to know what this difference is, in order that they may all be referred to some one standard. It is usual to refer the observations of all the staff of an Observatory to that of one of them. The average difference in the time of a star's transit over a wire as recorded by this observer and any other is called the Personal Equation of the latter.

57. *Clock-error determined.*

It has been said (Art. 32), that the clock should indicate $0^h, 0^m, 0^s$ at the moment of the passage of the first point of Aries across the meridian; and that the time of transit of any star across the meridian as shewn by the clock should give, when converted into angle at the rate of 15^0 to an hour, the R.A. of the star.

As it is impossible that the clock should for any length

of time be without error and rate, it is necessary to determine what these are, from time to time. For this purpose certain stars are observed, called *clock-stars,* whose R.A.s are known from the Nautical Almanac; the time of transit of any one of these stars will differ on any given day from that calculated from its R.A. This difference, corrected for the personal equation of the observer, gives the clock-error, as determined by this star. The mean of the clock-errors as found by observations of several stars is assumed as the actual clock-error.

The difference of clock-errors as determined on two successive days will give the *rate* of the clock.

58. *Galvanic (or chronographic) method of observing transits.*

In some observatories a method is adopted which renders it unnecessary for the observer to notice the position of the star at the commencement or end of a second. This is called the galvanic (or chronographic) method of observing transits.

A sidereal clock is fitted with an apparatus by which it causes a cylindrical brass barrel to revolve on its own axis, which is horizontal, in two minutes of time, while a frame moves parallel to its axis at the rate of $\frac{1}{10}$th of an inch to every revolution of the barrel. The motion of the clock is governed by a conical pendulum, which enables it to move uniformly without jerks, so that the barrel and the frame have each a perfectly uniform motion. The barrel is covered with a sheet of paper. To the frame are attached two electro-magnets; near each magnet is a lever, which, when attracted by the magnet, causes a pricking point to descend and puncture the paper which envelopes the cylinder. One of these magnets is connected, by the wires of a galvanic battery, with the clock, which completes the circuit at every second of time. By this means the commencement of each second is marked on the barrel by a puncture; the punctures so made being at equal distances along a spiral.

The wires of a galvanic battery connected with the other magnet pass through the transit-telescope. At the instant of the passage of a star across any wire, the observer touches a key, by which means the circuit is com-

pleted and the pricker punctures the paper on the barrel. By observing the position of the puncture thus made with reference to those made by the other pricker, the second and fraction of a second at the instant of transit are easily obtained.

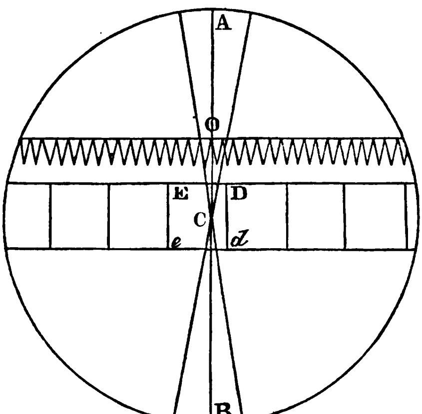
In this method of taking transits, the personal equation is much less than in the other, and depends solely on the observation by the eye of the instant of passage of the star over each vertical wire.

59. *Reading Microscope.*

Before describing the Mural Circle it will be necessary to explain the construction and use of the *Reading Microscope.*

The object of the Reading Microscope is to magnify the intervals between the divisions on the graduated limb of an instrument.

It consists of three lenses, of which one is the object-glass and the other two form the eye-piece; the object-glass is capable of adjustment by being screwed up towards the eye-piece or away from it. In the focus of the eye-piece is placed a spider-line micrometer; this micrometer con-



sists of a frame carrying an acute cross formed of two spider-lines, moveable across the field of view of the microscope by turning a micrometer-head attached to the

microscope. The middle of the field of view is marked in the figure by a line *AB*, parallel to the graduations of the instrument. The microscope is so placed that a part of the graduated limb is refracted by the object-glass to the focus of the eye-piece. In the figure, let *Dd*, *Ee* be successive graduations as seen by the microscope, one on each side of *AB*. And suppose the graduations on the rim to increase from left to right; then, since the microscope inverts, the graduations as seen in its field of view increase from right to left, or from *Dd* in direction *DE*. The micrometer-screw is turned so as to place *C*, the intersection of the cross accurately on *AB*; the micrometer-reading is taken for this position. The cross is then moved till *C* comes on *Dd*, and the reading taken again. The difference of the readings determines the interval between *AB* and *Dd*.

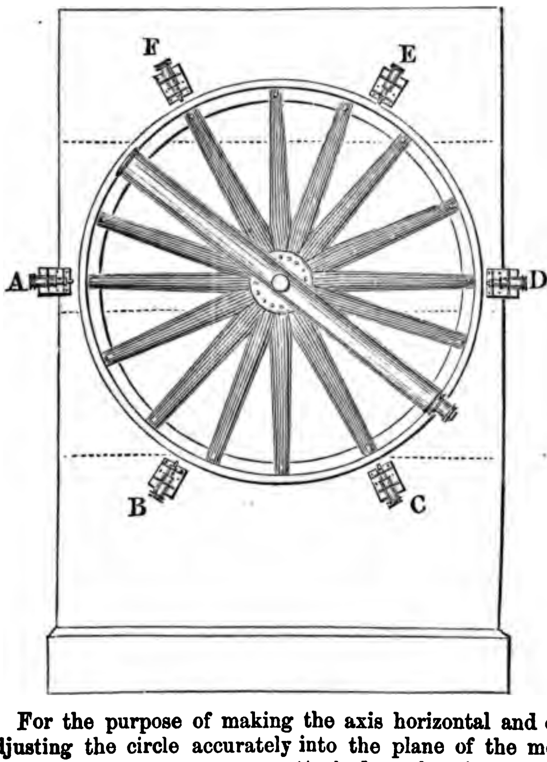
The micrometer-head is divided into 100 equal parts: and the pitch of the screw is such that five revolutions of the head will move the intersection of the cross-wires from coincidence with one graduation to coincidence with the next, supposing the part of the rim which is under to be at exactly the required distance from the object-lens of the microscope. One revolution of the screw will therefore indicate 1′ on the rim.

60. *Arrangement of the micrometer.*

In practice, the micrometer screw-head is so arranged that when the pointer indicates zero the intersection of the cross-wires is at the middle of the field of view, and the reading of the head increases as the cross-wires are moved towards *Dd*; thus the reading at *Dd alone* gives the distance of *Dd* from *AB*. The intersection of the cross-wires, whenever the pointer indicates zero, is directly under a notch cut in a plate fixed in the field of view of the microscope. There are therefore five notches for every interval of 5′ on the rim of the circle; above the notch at the middle of the field of view is a hole which indicates the position of *AB*. Thus the entire minutes are given by the number of notches between the cross-wires and the hole, and the fractions are given by the reading of the screw-head.

61. *The Mural Circle.*

The Mural Circle consists of a metallic circle firmly connected by conical radii, with a horizontal conical axis concentric with the circle. This axis is supported by a stone pier in which it is inserted, the plane of the circle being parallel to, and almost touching, the face of the pier, which coincides very nearly with the meridian.



For the purpose of making the axis horizontal and of adjusting the circle accurately into the plane of the meridian, there are two screws attached to the pier, which can give respectively a vertical and horizontal motion to the axis.

The object of the circle is to find the zenith-distances or N.P.D.s of stars at the instant of crossing the meridian. The rim is graduated very accurately from 0^0 to 360^0, the intervals between successive graduations being $5'$, so that 12 of them make up 1^0; the graduations are perpendicular to the plane of the circle.

A telescope is attached to the circle in such a manner that its line of sight is parallel to the plane of the circle.

When the telescope is directed to the zenith and then turned towards the *north* point of the horizon the readings under each microscope *increase.*

At the focus of the telescope are fixed 5 or 7 vertical wires and one horizontal wire, besides a micrometer horizontal wire moveable in altitude. The micrometer-head is divided into 100 equal parts. The telescope is firmly clamped to the circle, so that it is carried round with the circle in the plane of the meridian. The position of the line of collimation of the telescope with respect to the zero point of the graduations of the circle is immaterial. A pointer is attached to the pier in such a position that it indicates on the rim the number of degrees and the nearest five minutes of the N.P.D. of a star which is in the field of view and bisected by the fixed horizontal wire.

62. *Arrangement of the reading-microscopes.*

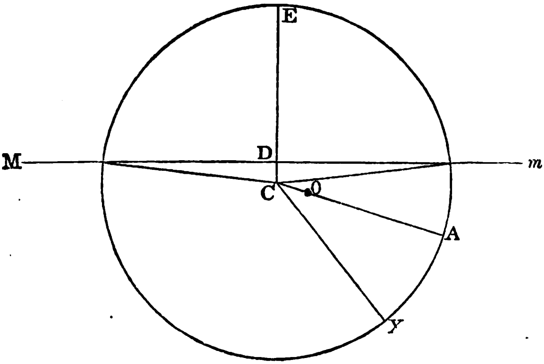
Six reading-microscopes are firmly attached to the stone pier, all looking directly on to the divided rim of the circle. They are at equal distances from each other, and consist of three pairs, the microscopes of each pair being diametrically opposite each other, so as to have a common line of sight as nearly as possible passing through the centre of rotation of the circle, in order that every point of the rim as it is brought under the object-glass of each microscope may be as nearly as possible at the same distance from it.

63. *Imperfect centering corrected.*

By this arrangement of the microscopes in pairs, and by taking the mean of all their readings, any error arising from *imperfect centering*, i. e. from the non-coincidence of the centre of the rim with the centre of rotation, is eliminated.

For, let Mm be the line of sight of any one of the pairs, C the centre of the graduated rim, O the centre of rotation.

Then CO produced meets the rim in a fixed point A.



Let X be the zero point of the graduations.

Draw CD perpendicular to Mm, meeting the rim in E.

Then, the mean of the readings given by the pair of microscopes is—supposing the circle to be graduated from X in direction XAm—$=\frac{1}{2}(\angle XCm + \angle XCM)$ $=\frac{1}{2}(\angle XCE - \angle ECm + \angle XCE + \angle ECM) = \angle XCE$. Now, suppose the circle to be turned about O through any angle, then every line fixed in the plane of the circle turns through this angle: thus OC and XC turn through the same; also CE is fixed in space, because Mm is. Hence the change in the $\angle XCE$ is equal to the angle through which the line OC has revolved, or to the angle through which the circle has rotated.

Thus, the difference of the means of the readings of all the pairs of microscopes for two positions of the circle gives accurately the angle through which the circle has been turned between those positions; or, which is the same thing, the angle between the two positions of the line of sight of the telescope.

64. *Mural circle adjusted.*

Although the plane of collimation of the circle-telescope should be pretty accurately in the meridian, yet this is not of so much importance as in the case of the transit instrument. For at the instant of crossing the meridian

the star is moving horizontally; its altitude, or its meridian zenith-distance, will therefore vary very slightly from the zenith-distance in any plane nearly coinciding with the meridian. The adjustments are performed by comparing simultaneous observations made with the circle-telescope and a transit instrument.

The axis of the circle is made horizontal by moving the screw which gives the vertical motion to the axis, until a star *near the zenith* passes the middle wire of the circle-telescope at the same time as it passes that of the transit instrument. It is made perpendicular to the meridian by moving the other screw until a star *near the horizon* transits the wires of the two instruments simultaneously.

65. *Value of a revolution of the micrometer determined by observation of the Sun.*

By turning the micrometer-screw of the telescope through one revolution, we change the position of the horizontal moveable wire, and thereby the position of the point in which it intersects the middle vertical wire. In order to estimate the value of a revolution of the micrometer, it is necessary to know what angle is subtended by two such points at the optical centre of the object-glass. This may be done by observation of the Sun: as soon as the Sun enters the field of view, the telescope is placed so that the fixed horizontal wire just touches the Sun's disc at its lowest point. The micrometer-wire is then brought, by turning the screw, into the position in which it just touches the Sun's disc at its highest point. After this has been done, the reading of the micrometer-head is taken; the difference of the readings for this position of the micrometer-wire and for coincidence with the fixed horizontal wire corresponds to the angle subtended at the object-glass by a diameter of the Sun's image, i. e. to the angular diameter of the Sun. And since this is given in the Nautical Almanac for noon on every day in the year, the number of revolutions and parts of a revolution corresponding to a given angle is known: from this the angle corresponding to one revolution is easily found.

66. *By two known stars.*

Another method is to take for two known stars similar

observations to those described above for the two limbs of the Sun. It may happen that the field of view is too small to admit the entire image of the Sun, in which case we must find two stars whose R.A.s are nearly equal, and whose difference of N.P.D. is small enough, and adopt this second method.

We now proceed to explain how observations are taken with the Mural Circle.

67. *Relation between Zenith-point and Nadir-point.*

The object of an observation of a star with this instrument is to find the zenith-distance of the star; and this will be given (Art. 63) by the difference between the circle-readings for the zenith and the star. It is necessary therefore, first of all, to know the circle-reading for the zenith: when this is known, by subtracting it from that for the star we obtain the zenith-distance of the star.

If the telescope could be placed so that the plane through the optical centre of the object-glass and the fixed horizontal wire were accurately vertical, the circle-reading would be that of the zenith-point if the object-glass were uppermost, and of the nadir-point if the eye-piece were so.

Now, the divisions under any microscope increase as the telescope, when pointing to the zenith, is turned through the North Pole (Art. 61) to the nadir. Hence, the Zenith-point will be found from the Nadir-point by subtracting from its reading 180^0.

68. *Nadir-point determined.*

By the following method the Nadir-point is readily found. A trough of mercury is situated immediately under the centre of the axis of the instrument. A Bohnenberger's eye-piece (Art. 51) having been substituted for the usual eye-piece, the object-glass of the telescope is directed towards the mercury, and moved until the reflexion of the spider-lines is seen in the field of view. It is then by a tangent-screw—a contrivance by which a very slight angular motion can be given to the instrument—moved until the reflexion of the fixed horizontal wire coincides with the wire itself. When this is the case, the plane through the wire and the centre of the object-glass

is vertical. It remains then simply to read the degrees and 5-minute spaces by the pointer, and add the mean of the readings of the micrometers for the divisions of the limb immediately preceding the middle notch in the field of view of the microscopes (Art. 60).

69. *Micrometer readings.*

Each microscope gives as its reading some number of minutes less than 5, being the number of complete revolutions of the micrometer, together with (since the micrometer-head is divided into 100 parts) two decimal places by the number of divisions on the head indicated by the pointer attached to the micrometer: a third decimal place is obtained by estimating in 10ths of a division the interval which this pointer indicates from the nearest preceding division.

In taking the mean of the micrometer readings, it is convenient to take the mean of the integral minutes separately, and add them to the mean of the fractional parts.

For the fractional parts have to be added, their sum divided by 6, and then multipled by 60 to get the seconds. This amounts to simply adding, and in the sum shifting the decimal place; the result of this part of the reading is thus expressed at once in seconds and tenths of a second.

70. *Zenith distance of a star determined.*

Having thus obtained the reading for the Nadir-point, we have, by subtracting 180°, the reading for the Zenith-point.

To find the Zenith-distance of any star, the telescope is set by the pointer approximately to its N.P.D., so as to bring it into the field of view. By the tangent-screw the horizontal fixed wire is made to bisect the star. The microscopes are then read, and the rest of the operation is precisely the same as for finding the Nadir-point. The reading for the Zenith, obtained by subtracting 180° from that for the Nadir, is subtracted from the reading for the star, and the result is the Zenith-distance of the star.

71. *Horizontal-point determined.*

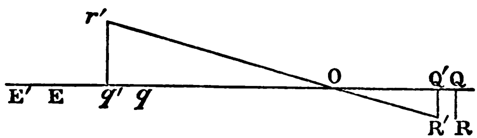
Sometimes, instead of the Nadir-point, the horizontal point is found. This is done by two observations of a star,

one of them directly, and the other after reflexion from the mercury. The line of collimation is at the same angle above or below the horizon according as the star or the star's reflexion is observed. The semi-sum of the readings will therefore be the reading when the line of collimation is horizontal.

72. *Effect of temperature on the image of the rim.*

It has been assumed in what has preceded, that 5 revolutions of the micrometer-head *exactly* move the intersection of the cross-wires over one 5-minute space of the rim. This is not the case, however, even if the micrometer-screws are made with perfect accuracy; for changes of temperature, by expanding and contracting the circle, affect the readings of the microscope, as we proceed to shew.

Let EQ be the optical axis of one of the microscopes,



meeting the rim of the circle in Q, E the optical centre of the inner eye-lens, O of the object-lens, and q the principal focus of the eye-piece.

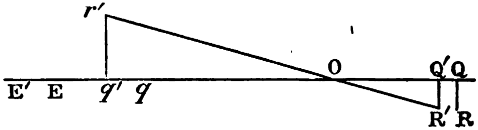
Suppose, now, that the microscope has been accurately adjusted, so that q coincided with the image of Q formed by the object-lens. Again, suppose by an increase of temperature the rim to expand and Q to be brought nearer to O. Since O is a convex lens, the geometrical foci move in the *same* direction. Hence as Q moves nearer to O, its image moves farther off, or nearer to E, and is no longer at the principal focus of the eye-piece, and is, therefore, not in a position for distinct vision.

In order to make the rim distinctly visible, the eyeglass of the microscope is shifted so that the image of the rim may be in its focus.

73. *Run of the microscope; effect of temperature on the runs.*

Suppose the rim to have expanded by an increase of temperature, so that Q' is the new position of the point Q of the rim, and q' of its image after refraction through

the object-glass; then q' is nearer to E than q was: E will therefore have to be moved to E', making $EE' = qq'$. If, now,



QR, the interval between Q and the next division on the rim, be, by the expansion of the rim, brought to $Q'R'$, the angle QOR is increased; hence the image $q'r'$ of $Q'R'$ subtends a greater angle at O than the image of QR; and it is at a greater distance from O; it is therefore increased.

The number of revolutions, and parts of a revolution, corresponding to a five-minute space on the rim is called the *run* of the microscope.

Thus, expansion of the rim increases the runs of the microscopes; and, similarly, contraction of the rim diminishes them.

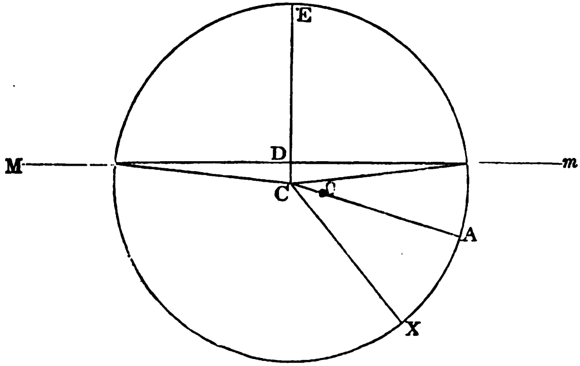
74. *Error of runs determined.*

Since the runs are liable to variation, it is necessary from time to time to determine the *error of runs*. The mode of doing this is, to add the runs given by all the microscopes, take their mean, and subtract the result from $5'$. Having done this, we can, by a proportion, find the correction for runs to be applied to any number of minutes or seconds given by the microscopes. The mean of the corrected readings of all the microscopes is the true reading of the circle free from error of runs.

75. *Mean of the readings of the microscopes unaffected by the effect of imperfect centering on the runs.*

In Art. 63, it was assumed that the microscopes, whose common line of collimation points to M and m, give the correct readings in all positions of the circle. This will be the case if M and m are each precisely at the right distance from the object-lens. If not, the reading of each is affected with an error of runs. We shall assume that the microscopes at M and m have been read in a certain position of the circle, in which either they have no error of runs, or this error has been found for each and the cor-

rected readings of the microscopes taken as the true readings. If the circle be now turned about O through any angle, the *new* position of the circle will cut Mm in two points, which will not in general coincide with M and m.



Now the circle may be conceived to be put into its new position by moving it, first parallel to Mm, so as to bring its centre on to a line through its new position perpendicular to Mm, then perpendicular to Mm till the centre coincides with its new position: the rim of the circle is thus made to coincide with its new position, and must be turned through an angle about its centre C to bring each point of it into the position into which it is brought by the rotation about O.

The first of these three motions moves M and m in the same direction along Mm through equal small spaces. The reading of one microscope will therefore be as much increased as that of the other is diminished, and the mean unaltered. The second motion, since the circle is very nearly perpendicular to Mm at M and m, does not sensibly alter the positions of the points in which the circle cuts Mm. And the third motion clearly does not affect these points. Thus the rotation about O produces no error of runs in the mean of the microscope readings.

76. *The Transit Circle.*

The Transit Circle consists of a Transit Instrument,

perpendicular to the axis of which is rigidly attached, near one end of the axis, a Circle supported by and rotating about the axis of the Transit Instrument. By this combination the R.A. and N.P.D. of a star may both be found by observations made by the same observer.

The Circle is usually graduated, not on its rim, as is the case of the Mural Circle, but in the plane of the circle; the graduations are made on a ring of metal contiguous to the rim, and point towards the centre of the circle. The microscopes, arranged in pairs as for the Mural Circle, are attached to one of the piers, and have their axes perpendicular to the graduated ring. Parallel to the Circle whose graduations are read by the microscopes, and similarly situated at the other end of the axis, is another graduated circle, also revolving with the instrument, and read, for the purpose of setting it approximately to a given N.P.D., by a microscope or a vernier attached to the other pier.

The graduations being in the plane of the Circle, it is easy to see that any small expansion or contraction of the circle will very slightly affect their distance from the object-lenses of the microscopes. The consequence is, that the error of runs is in this case very small.

The errors of adjustment of the Transit Circle are of course the same as for the Transit Instrument. In the case of the Transit Circle, however, it is not expedient, nor always possible, to reverse the axis; nor is it adapted for the application of a spirit-level.

We are therefore unable to determine the error of collimation by collimating marks, and the error of level by the spirit-level.

77. *Error of collimation by collimating telescopes.*

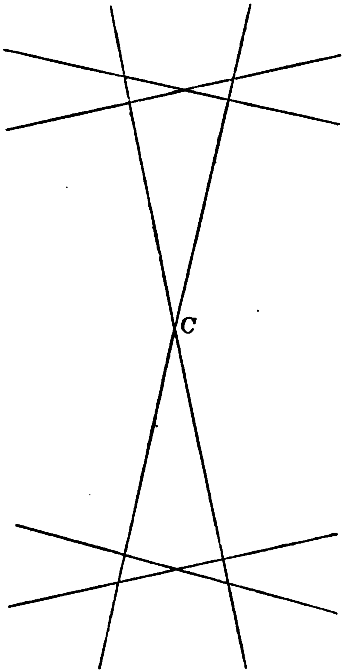
For the purpose of determining the error of collimation the following process is employed.

Two small telescopes are placed on piers north and south of the Circle, with their optical axes horizontal and in the same straight line: this line being nearly in the plane of collimation of the Circle-telescope and passing nearly through the centre of rotation of the Transit Circle; they are called *collimating-telescopes*, or *collimators*.

Their object-glasses are turned towards each other, and therefore towards the instrument. At the principal focus of each of them is a system of wires forming a square, two of the sides of which are nearly vertical, and the other two nearly horizontal. The nearly vertical wires are inclined to the vertical at about the same angle in each telescope and on opposite sides of the vertical.

The error of collimation of the Circle-telescope is determined by the collimators in the following manner.

78. *Setting the collimators.*



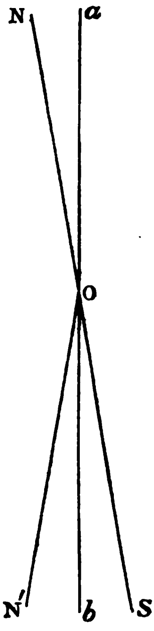
The tube of the circle-telescope being placed horizontal, the Circle is raised, to allow the field of view of each collimator to be seen in the field of view of the other.

The system of wires in one of the collimators is capable of motion by a micrometer in a horizontal direction: one of the nearly vertical sides of its square may by this means be placed so as to bisect the image of one of the sides of the square of the other,—as at the point C in the figure. This can be done with great accuracy, for it is clear that a very small motion of either wire makes a perceptible difference in the ratios of the right-angled triangles which have a common angular point at C. The reading of the micrometer which gives the horizontal motion is then taken.

This operation is repeated several times, and the mean of the readings taken as the true reading. The micrometer-head is then turned to this reading. In the position in which the wires have been thus placed, the line of sight of the middle point of the vertical wire is the same straight line in each collimator.

79. *Reading for collimation determined.*

The circle having been now replaced, the object-glass of its telescope is turned towards the north collimator. The *wire-frame* is moved by the micrometer-screw, till the middle wire bisects the nearly vertical wire of the collimator. The micrometer-reading is then taken. The telescope is next turned to the south collimator, and its vertical wire bisected. The micrometer is again read. The mean of the two readings will give the reading for the line of collimation.



For, let NS be a line through O, the optical centre of the circle-telescope, parallel to the common line of sight of the collimators; ab the line of collimation of the circle-telescope. Then ON is the line of sight at the first reading; and if we make $\angle N'Ob$ equal to $\angle NOa$, ON' is the position of ON when the telescope is turned round for the second reading. Since $\angle N'Ob = \angle NOa = \angle SOb$, the mean of the readings for ON (or ON') and OS is the reading for Ob, i. e. for the line of collimation.

80. *Error of level by Bohnenberger's eye-piece.*

The error of collimation having been found as above, the error of level can be determined by Bohnenberger's eye-piece, the description and use of which we have already given (Art. 51).

81. ***Moveable horizontal wire; value of the readings of its micrometer-screw determined.***

The value of a reading of the micrometer-screw attached to the moveable horizontal wire at the eye-end of the telescope is found by means of one of the collimators in the following manner. The head of the screw is placed at a given reading; e.g. at the zero of the graduations. The telescope is then directed to one of the collimators—say the north—and moved till the image of the horizontal wire passes through any one of the points of intersection of the wires of the collimator.

The microscopes are then read, and the mean of the readings taken.

The micrometer-screw is then turned through any angle and the reading of the head taken. The telescope is then moved by the tangent-screw till the horizontal wire is made by the motion of the telescope to pass through the same point of the collimator-wires as before. The microscopes are read again. The difference between the means of the microscope-readings for the two positions of the telescope is the angular value of the interval between the two positions of the horizontal wire, the micrometer-reading for which has been taken: from this the angular value of a revolution of the screw can be easily calculated.

82. *Observing the Sun, Moon, or a planet.*

If the Sun be the object observed, the telescope is placed so that the *fixed* horizontal wire just touches the upper limb of the Sun, and the telescope being kept in that position, the *moveable* horizontal wire is placed just in contact with the lower limb. Half the difference of the readings of the micrometer gives the angular radius of the Sun. The mean of the microscope-readings gives the N.P.D. of the upper limb. And the sum of these two results is the N.P.D. of the Sun's centre.

The time of transit of the Sun's centre over each of the vertical wires is the mean of the times of first and last contact of the disc with the wire.

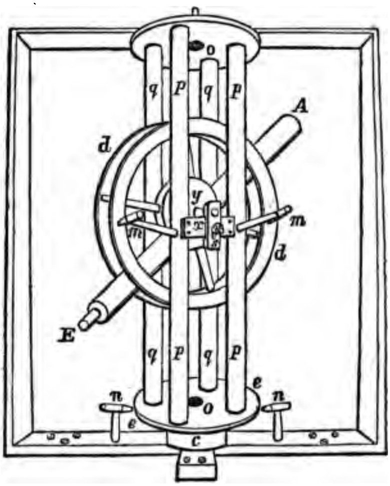
In the case of the Moon or a planet, the illuminated limb alone can be observed, either for N.P.D. or for

transit. The N.P.D. and time of transit of the centre can be calculated from this by applying corrections depending on the angular diameter and an approximate N.P.D. of the centre.

83. *The altitude and azimuth instrument.*

This instrument, also called the altazimuth, is useful where observations are required of an object at times when it is not on the meridian. It may be used to observe an object in any position; so that observations can be taken with it very much more frequently than with instruments in the plane of the meridian.

It consists of a graduated vertical circle, to which is rigidly attached a telescope; the axis of rotation of the



telescope is horizontal and passes through the centre of the circle, and terminates in pivots supported by Y's; the telescope is thus capable of revolving together with its attached circle in a vertical plane. The Y's of the telescope are attached to a frame, which is capable of revolving about the vertical diameter of the circle.

The instrument is placed upon a stone pillar on which is firmly fastened a horizontal graduated circle whose centre is on the vertical diameter of the vertical circle.

To the outer surface of the revolving frame are attached four vertical reading microscopes by which the horizontal circle is read, and four horizontal microscopes to read the vertical circle. The diameters of the cirlles are a little greater than that of the frame, so that their graduations—which are in the plane of each circle—can be read by the microscopes.

At the principal focus of the telescope is fixed a frame carrying six vertical and six horizontal wires.

84. *Observations with Altazimuth. Adjustments.*

We are able with this instrument to take observations of two kinds; either we may determine by the sidereal clock the sidereal time of transit over the mean of the vertical wires, and read the horizontal circle, by which means we obtain the azimuth and sidereal time when a star crosses the mean of the vertical wires : or we may determine the sidereal time of transit over the mean of the horizontal wires, and read the vertical circle, by which we obtain the altitude of the star and the sidereal time when it crosses the mean of the horizontal wires.

The inclination of the horizontal axis to the horizon is ascertained by a spirit-level placed with its feet on this axis, as in the Transit Instrument.

The inclination of the vertical axis to the vertical is found by attaching a spirit-level to the frame and revolving the frame through 180°. By the motion of the bubble in the level the inclination of the vertical axis can be determined.

85. *The Equatorial**.

The general principle of this instrument is the same as that of the altazimuth, the chief difference being that the

* The student will find at the end of the book a representation of a model of an Equatorial, for which, as also for that of the Altazimuth on the opposite page, the author is indebted to the kindness of Professor Challis.

axis which is vertical in the altazimuth is placed in the direction of the Earth's polar axis in the Equatorial; it is called the *polar axis* or *hour-axis*. The horizontal circle thus becomes a circle parallel to the equator. This circle is called in the equatorial the *hour-circle;* the circle which corresponds to the vertical circle of the altazimuth is called the *declination-circle;* and the axis about which this circle revolves corresponds to the horizontal axis, and is called the declination-axis. If the instrument be turned about the polar axis through any angle, the line of collimation of the telescope will maintain the same inclination to the polar axis throughout the motion, and will therefore describe a portion of a small circle in the heavens about the pole. If, therefore, the circle be turned in the same direction as the diurnal motion of the stars, uniformly and at such a rate as to complete a revolution in one sidereal day, and the telescope be placed so as to have initially a given star in its field of view, that star will remain in the field of view. In this consists the peculiar advantage of the Equatorial; the motion, above described, of the hour-circle can be secured by appropriate machinery worked by clock-work; the observer is thus able, without moving the instrument himself, to take any number of observations of a heavenly body in the course of a night.

The hour-circle is graduated to read the angle described about the polar axis by the declination-circle which passes through the line of collimation of the telescope, and the declination-circle is graduated to read the north polar distance of any star in the centre of the field of view.

The hour-circle is usually divided into hours, minutes, and seconds, instead of degrees, minutes, and seconds: 24 hours corresponding to 360°, or 1 hour to 15 degrees. Both the circles are capable of motion about axes perpendicular to them, and are read by fixed microscopes or verniers.

At the principal focus of the telescope is fixed a frame containing a system of wires parallel to the declination-circle, and two wires perpendicular to these, or parallel to the hour-circle: the latter wires are moveable by a micrometer with two heads, each head moving one of the wires.

86. *Uses of the Equatorial. Differential observations.*

The Equatorial is used for objects for which it is necessary to have a great number of observations in a given time. For instance, in order to determine correctly the path of a comet it is necessary to take a great number of observations of it while it is in sight. The Equatorial supplies a means of doing this.

From the want of symmetry with respect to the vertical and consequent instability of the instrument, which is a necessary consequence of the arrangement of its different parts, *direct* observations of N.P.D. with it are very inferior in accuracy to those made with meridian instruments.

The Equatorial is therefore always employed for taking *differential* observations: i.e. for observing the R.A.s and N.P.D.s of two objects in the field of view, and taking their *differences*. The errors from imperfect adjustment will affect two near objects very nearly equally, and the differential results will therefore be very nearly accurate.

Thus, by the micrometer-screw we can move the two wires which are parallel to the hour-circle till they just touch a planet or the Sun on both sides; the number of turns of the micrometer necessary to bring the wires from this position into coincidence will enable us to determine the angular diameter of the body.

We may also in observing a comet compare it with a fixed star in its neighbourhood. The place of the star being assumed to be known correctly, we obtain by a series of such observations the motion of the comet with reference to it, and the path of the comet during the period of its being observed becomes known.

87. *Conditions of perfect adjustment.*

The instrument, if in perfect adjustment, will satisfy the following six conditions.

(1) Its polar axis must be in the meridian.

(2) The inclination of the polar axis to the horizon must be equal to the latitude of the place.

(3) The axis of revolution of the declination-circle, called the *declination-axis*, must be at right angles to the polar axis.

(4) The line of collimation of the telescope must be at right angles to the declination-axis.

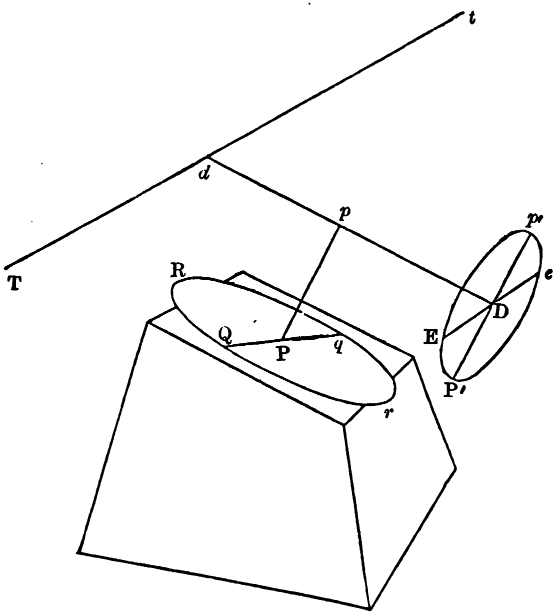
(5) The declination-circle must read 0° when the line of collimation of the telescope points to the pole.

(6) The hour-circle must read 0^h, when the declination-circle is parallel to the meridian, or when the declination-axis is horizontal.

We have no space to devote to a description of the methods by which these conditions are secured. The student will find a full account of them in Chambers's *Handbook of Astronomy*, pages 341—344.

88. *Readings of the hour-circle and of the declination-circle give the motions of the optical axis in R.A. and N.P.D.*

In the figure, *RQr* is the hour-circle; *Pp*, the axis about



which this revolves, is the polar axis, or *hour-axis*. Perpendicular to this and revolving about Pp is the declination-axis Dd. The circle Ee perpendicular to the declination-axis is the declination-circle. Tt is the optical axis of the telescope: Tt, being parallel to the plane of the declination-circle, is perpendicular to Dd. Also the declination-circle is perpendicular to the hour-circle and parallel to the polar axis. Draw $P'p'$, in the plane of the declination-circle, parallel to the polar axis.

Let a plane parallel to the declination-circle Ee meet the hour-circle Rr in Qq. Then, as the clock-work moves the hour-circle round, the whole instrument is carried round Pp; and Qq is carried by the circle through angles equal to the angles described by the declination-circle through Tt the optical axis of the telescope.

Again, if the motion of the hour-circle were stopped, and the declination-circle turned round, it would carry Tt through an angle equal to the angle described in its own plane by the declination-circle. If, then, Ee be a line in the plane of the circle parallel to Tt, the angle through which it revolves by the revolution of Ee in its own plane is the angle through which Tt has been moved in N.P.D.

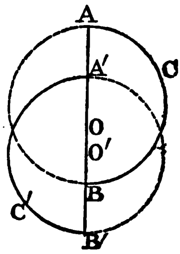
The whole instrument is supported by a pier, on a face of which the hour-circle Rr rests.

89. *The Heliometer*.

The Heliometer is simply an Equatorial with a divided object-glass. The object-glass is divided into two equal parts by a plane through its optical centre and the optical axis of the telescope. There is an apparatus by which these two parts may be made to slide past each other. When the centres of the two halves coincide, there is a complete object-glass, which will give but one image of a star. When the centres are separated, each will produce an image of a star in the same position as it would occupy if it were produced by a whole object-glass with its centre in the position of that of the half.

Thus let O, O' be the centres of the two halves ACB, $A'C'B'$, and imagine each semicircle completed by the dotted lines. Then, the images of a star produced by the two halves will be at equal distances from O and O', on

the lines drawn through O, O' in the direction of the star. They will therefore be at a distance OO' from each other in a line parallel to OO' at the principal focus of the telescope.



Attached to the screw by which the half lenses are separated, is a graduated head read by a pointer, as in the ordinary micrometer. The line AB is capable of being turned round into any required position by a motion of the object-glass in its own plane. In order to determine the value in angle of a reading of the micrometer the telescope is set to point to a known star, and the object-glass is turned so as to make AB coincide with the direction of the diurnal motion of the star; the micrometer-head, being turned through one revolution, separates the halves of the object-glass and gives two images of the star. The times of transit of these two images over a wire perpendicular to AB being observed, their difference is the time occupied by the star in describing an angle equal to that subtended at the centre of either half of the object-glass by a line at the focus equal to OO'; also the angle described in a given time by the star in its diurnal motion is known: thus the angular value of OO' is known. Hence we know the value in angle of the separation of the images corresponding to any number of turns or parts of a turn of the screw.

90. *Observations of stars with the Heliometer.*

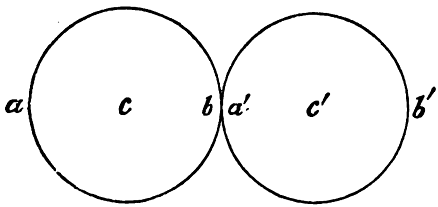
Suppose now, for example, it is desired to measure the angular distance between two stars S_1, S_2, which are near together. AB is placed so as to coincide with the direction of the line S_1S_2: and the halves of the object-glass having been adjusted so as to give only one image of each star, the micrometer-screw is then turned till each star has moved through the space S_1S_2. Since the motion is parallel to AB, it is in the direction S_1S_2, and therefore the image by O of one star will coincide with the image by O' of the other; the screw being therefore turned till this takes place, the reading determines the required angular interval between the stars.

The instrument may also be used for measuring the diameters of planets and of the Sun: and it is from the application of it in this manner to the Sun that it owes its name.

91. *Observation of the Sun.*

If the Sun be the object observed, each point of its disc will give two images in a straight line parallel to OO', the distance between them for each pair being equal to OO': hence the whole disc, by the separation of the halves of the object-glass, is moved through OO' in a direction parallel to OO'.

If the reading of the micrometer be taken when the images just touch as at ba', and the screw be then turned till they just touch on the other side, as at a and b'; the difference of the readings in the two positions will correspond to a motion of one centre relatively to the other



equal to twice cc', the distance between the centres when the images are in contact, that is, to twice the Sun's diameter. Half the difference, therefore, determines the Sun's angular diameter.

92. *The Zenith Sector.*

There are several forms of this instrument, the use of which is to measure differences of zenith-distances for objects near the zenith with great accuracy. We shall very briefly describe one of the simplest forms.

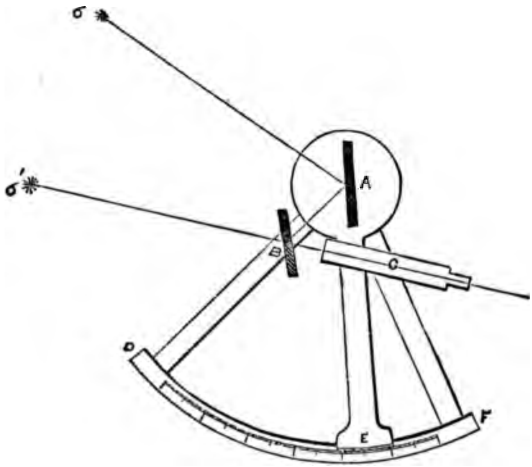
It consists of a long telescope rotating about a horizontal axis which is very near the object-glass. The eye-end of the telescope is nearly vertically under the object-glass, and the telescope is capable of motion about its axis through a small angle in either direction. Near the eye-

piece is attached to the telescope a small circular arc whose centre is on the axis of the sector and plane perpendicular to the axis; this arc is graduated on both sides of its middle point. A plumb-line, hanging from a point on the axis, passes close in front of the graduated arc: behind the plumb-line is a reading-microscope with its optical axis parallel to the axis of rotation of the telescope, by which the graduation of the arc, at the point where the plumb-line is projected on it, is read.

93. *Hadley's Sextant.*

There is another instrument which, though not used in *fixed Observatories*, is of such extensive use for nautical purposes, that a short description of it cannot properly be omitted. It is called *Hadley's Sextant.*

The principle on which this instrument is constructed, depends upon a proposition proved in Optics, viz. that



when a ray of light is reflected at two mirrors, the angle of deviation is equal to twice the angle between the mirrors.

The figure represents Hadley's Sextant; *DF* is a portion of a graduated rim, the graduation, as the name im-

ports, extending usually to about one-sixth of a circumference, but sometimes to more. AE is a moveable radius of the circle, which carries with it a small mirror A of silvered glass at one end, and a vernier at the other. B is a piece of glass silvered over half its surface, and is so fixed that when the reading of the vernier is zero, the surfaces of the mirrors A and B are parallel. C is a small telescope attached to the instrument, and so arranged that its axis passes through the line of division between the silvered and unsilvered parts of the glass B.

To find the angle subtended by the line joining two objects σ, σ', let the instrument be held in such a position, and the moveable radius so adapted, that the image of σ as seen by reflexion at the two mirrors, coincides with that of σ', as seen by direct vision; then the angle between the two objects will be twice the angle between the mirrors, or twice the arc between the pointer of the vernier and the zero point of the instrument, since when the mirrors were parallel the reading of the vernier was zero. Consequently, if we graduate the arc DF in such a manner that every degree of its arc is marked as two degrees, the reading given by the vernier will be the angle required.

By means of this instrument the angular distance between two objects may be ascertained with sufficient exactness for nautical purposes; and the altitude of a heavenly body may be found by holding the instrument with its plane vertical, and moving the radius AE into such a position that the reflected image of the body may, by a slight tilt of the sextant, just graze the horizon.

In practice, the reading of the vernier will not be accurately zero when the mirrors are parallel; the reading which it actually gives in that case is called the *index-error*. The index-error is determined by taking the reading of the vernier when the image of a distant object after reflexion at A and B is made to coincide with its image viewed directly.

CHAPTER III.

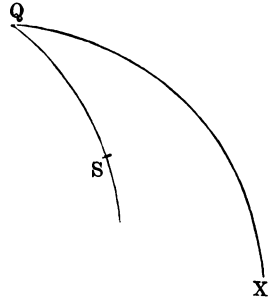
ON THE METHOD OF REFERRING THE POSITIONS OF THE HEAVENLY BODIES TO CERTAIN PLANES AND POINTS.

94. *Principle of the method.*

We have explained in Chapter I. how, by comparing from time to time the position of the Sun, Moon, or a planet among the fixed stars, its apparent motion may be traced. It is evident that the same result may be arrived at by finding the positions both of the fixed stars and of the moving bodies with reference to certain planes and points on the celestial sphere. If these planes and points be such that the positions of the fixed stars with reference to them do not vary, the directions of the planes and points themselves are invariable: if not, the change of the positions of the fixed stars with respect to them will determine their motions. These motions, if there be any, may be allowed for, and the positions of all bodies can then be referred to initial positions of the planes and points: and the motions of those bodies which do not retain an invariable position with respect to these initial positions may be found. This is the method by which the motions of the heavenly bodies are determined in practice.

We will now shew how the positions of bodies with respect to the planes and points to which they are commonly referred are determined, and how from the positions with reference to one set the positions with respect to others may be deduced.

95. *Position of a body referred to a point and a great circle passing through the point.*



Let Q be any known fixed point (Art. 19) on the celestial sphere, and QX an arc of a known fixed great circle. Then, the position of any body S is known, if SQ, the angular distance from Q, is known, and the angle SQX, which the plane of SQ makes with that of QX.

96. *The position of a body is known when its N.P.D. and R.A. are known.*

If Q be the north pole, and QX pass through the first point of Aries, SQ is the N.P.D.: and the angle SQX is the R.A. of the star. Thus the position of a star becomes known when its N.P.D. and R.A. are known.

97. *Determination of the position of the pole; and of the N.P.D. of a body.*

To determine the position of the pole we require the position of the meridian at the place of observation, the position of the zenith, and the value of ZP the co-latitude.

The position of the meridian is determined best by the pole-star. A telescope, moveable about a horizontal axis, is placed in such a position as to have the pole-star in its field of view. The superior and inferior transits are observed and the interval of sidereal time found. Suppose the inferior transit to follow the superior by *less* than 12 hours; the plane of collimation is then too much to the *left* of the north pole; the horizontal axis may then be shifted, and the observation repeated. By a sufficient number of repetitions of the process, the plane of collimation can be brought pretty accurately into the plane of the meridian, and the deviation may be calculated from the observed interval of time between the two transits.

The method of determining the position of the zenith has been described in Chapter II.

The latitude of the place of observation is equal to the

altitude of the pole (Art. 15); and this is equal to the semi-sum of the altitudes of a circumpolar star at its two transits (Chap. x.): hence ZP, the co-latitude, may be found by observations of circumpolar stars with a meridian circle. We thus know the position of the meridian and of the pole.

By subtracting the reading for the pole from the reading for the star, we determine the N.P.D. of the star.

98. *Determination of the position of the first point of Aries; and of the R.A. of a body.*

To determine the R.A. we require the position of the meridian through the first point of Aries.

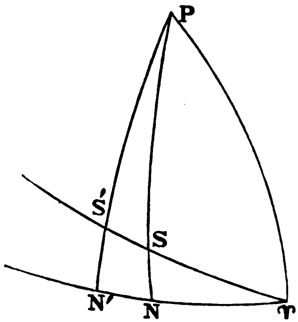
When the first point of Aries is on the meridian, the sidereal clock should indicate 0^h, 0^m, 0^s; and the angle which the meridian through it makes with the meridian of the place is known at any instant if the correct sidereal time is known (Art. 33). To find the position of the first point of Aries is, therefore, the same thing as to find the error of the sidereal clock, or the time indicated by it when the first point of Aries is on the meridian.

When the sun is near an equinox its N.P.D. is changing most rapidly; if therefore two observations be taken of the Sun near the time when its declination vanishes, the difference between the two N.P.D.s found will be tolerably large, so that any small errors of observation will be less important than they would be in other parts of the orbit. For this reason observations taken near an equinox are the most favourable for determining the positions of the points of intersection of the ecliptic and equator.

Suppose, now, observations to be taken of the Sun just after passing through the first point of Aries: the difference between the clock-times of transit of the Sun and a fixed star gives the difference of R.A. of the Sun and star. In consequence of the motion of the Sun this will vary from day to day. The difference between the results given on two successive days gives the motion in R.A. of the Sun in the interval. The N.P.D. of the Sun can also be found by the Transit Circle at each observation. And from these data the position of the first point of Aries is known.

For in the figure, let $\Upsilon SS'$ be the arc of the ecliptic

meeting the equator in ♈, the first point of Aries. And let PSN, $PS'N'$ be the declination-circles through the



centre of the Sun on the two days. Then, the motion in R.A., determined as above, gives $\angle NPN'$, or the arc NN': and the declinations NS, $N'S'$ are known from the N.P.D.s. Thus NN', NS, $N'S'$ being known, the position of the plane through the centre of the sphere and SS' is known, and all the parts of the spherical triangle ♈$S'N'$ can be found. Thus we can determine ♈N', the R.A. of the Sun when at S'; the difference between this and the R.A. as inferred from the time given by the sidereal clock is the error of the clock, or the sidereal time indicated by it at the transit at the first point of Aries.

To find the R.A. of a star, subtract the clock-time of transit of the first point of Aries from the time of transit of the star; if this difference, expressed in hours, be t, the R.A. in degrees is equal to $15t$.

Obs. From the geometry of the figure we may also calculate the $\angle S$♈N or the obliquity.

Though two observations are theoretically sufficient to determine the position of the first point of Aries and the obliquity, they would practically only give a first approximation. By a series of observations of this nature the approximate value first deduced may be still further corrected; and by taking the mean of the results of a considerable number of observations a very accurate value is obtained.

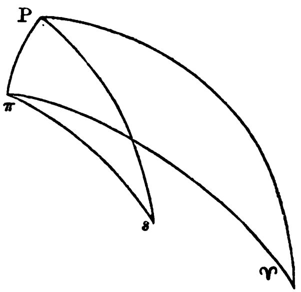
5—2

99. *Motion of the first point of Aries detected.*

If the R.A.s given by determinations of the positions of the equinox in different years be compared with each other, it is found that the R.A. of each star increases regularly by a small amount every year. Hence the direction of the first point of Aries is not fixed (Arts. 18, 94); its motion among the stars is retrograde, i.e. from east to west, or in a contrary direction to that of the Sun. The Sun will therefore pass through the vernal equinox earlier every year than if this point were fixed. The motion of Aries is, for this reason, called the *Precession* of the Equinoxes. It is discussed in Chapter VI.

100. *Position of a star referred to the pole of the ecliptic, and a great circle joining it with the first point of Aries.*

The position of a star being known with respect to the pole of the equator and the first point of Aries, its



position with reference to the pole of the ecliptic and the first point of Aries can be deduced.

For, let P be the pole of the equator, π of the ecliptic. Join ΥP, $\Upsilon\pi$: thence since Υ is in both planes of which P and π are poles, $P\Upsilon$ and $\pi\Upsilon$ are both quadrants, and Υ is therefore the pole of the great circle $P\pi$.

Let s be a star: join sP, $s\pi$. Then the R.A. of the star is the angle $sP\Upsilon$, which is the complement of $\angle sP\pi$; and the N.P.D. sP, and the obliquity $P\pi$, are known.

Thus, in the spherical triangle πPs, πP, Ps and the

$\angle \pi P s$ are known. The other parts of the triangle can therefore be found. Hence πs and the $\angle s\pi P$ are known.

And the angle $P\pi\Upsilon$ is a right angle; hence the angle $s\pi\Upsilon$ is known.

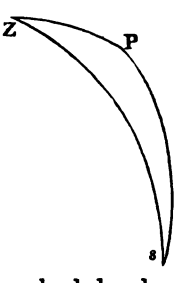
Thus the position of s being known with respect to P and $P\Upsilon$ we can deduce πs and $s\pi\Upsilon$ which determine its position with respect to π and $\pi\Upsilon$.

101. *Latitude and longitude of a star.*

DEF. If πs be produced till it meets the ecliptic, the angular distance of s from the ecliptic measured along it is called the *Latitude* of the star; and πs, which is its complement, is called the *Co-latitude*. And the angle $s\pi\Upsilon$ is called the *Longitude* of the star. A star's co-latitude and longitude determine its position with reference to the ecliptic and first point of Aries in precisely the same manner as the N.P.D. and R.A. do with respect to the equator and first point of Aries.

It is found that the longitudes of all stars increase from year to year, while the latitudes are not altered. This shews that the ecliptic is fixed, and that the motion of Aries is due to a motion of the equator.

102. *Position of a star with respect to the ecliptic deduced from observations with an altazimuth.*



Let Z be the zenith, P the pole, s a star. Then ZP is the co-latitude of the place; and if, with an altazimuth, the ZD. Zs be observed, and the azimuth PZs; these, with ZP, are sufficient to determine all the parts of the triangle ZPs. Thus Ps, the N.P.D., and $\angle ZPs$, the hour-angle, are known. Also the sidereal time gives the hour-angle of Υ: hence the R.A. and N.P.D. of s are known. These being known the latitude and longitude of the star may be deduced, as explained above.

We have thus shewn how, from the results of observations with the Transit Circle, or the Transit Instrument and Mural Circle, the Altazimuth, or the Equatorial, the posi-

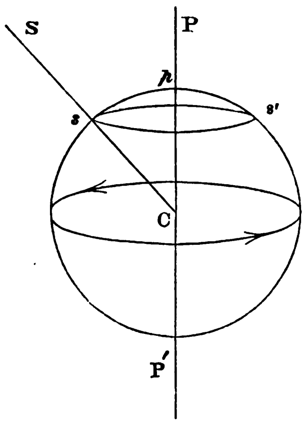
tion of a star or other celestial body, may be deduced with reference finally to the ecliptic; also, how the positions of the poles of the equator and ecliptic, and of the equinoxes, with reference to the fixed stars, may be found from time to time; and hence, how the motions of these points among the fixed stars, and thus the actual variations in the directions of the planes of the equator and ecliptic in space, may be determined.

CHAPTER IV.

ON THE PHENOMENA DUE TO THE ROTATION AND ORBITAL MOTION OF THE EARTH.

103. *Apparent rotation of the heavens accounted for by the rotation of the Earth.*

In the figure PP' is the axis, C the centre of the Earth: the diurnal rotation is performed by the Earth in



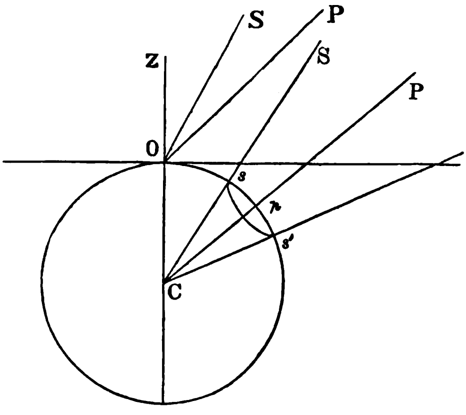
the direction of the arrow-heads: P and P' are the north and south poles respectively: S the position of any star. In the course of a day the Earth will have completed a rotation about PP': in the meantime the centre of the Earth has been moving over a small portion of its orbit round the Sun. The line CS, however, joining C with any star S,

has in this time been moving to all intents and purposes parallel to itself; PP' the axis of the Earth also moves parallel to itself; thus $\angle PCS$ has not changed. Hence as successive meridians of the Earth are made by its rotation to pass through S, the point s in which CS meets the Earth is always at the same distance from the pole p: and thus by the uniform rotation of the Earth the point s in which CS meets the surface of the Earth, describes uniformly a small circle parallel to the equator, in the opposite direction to that of the rotation.

104. *Stars appear to move in portions of circles parallel to the equator.*

We now proceed to shew from this how the star will seem to move to a spectator on the Earth's surface.

In the figure, let O be the position of the observer. Produce CO indefinitely to Z; then Z is the observer's



zenith; S any star when the plane of the meridian passes through it, and s the point in which CS meets the Earth.

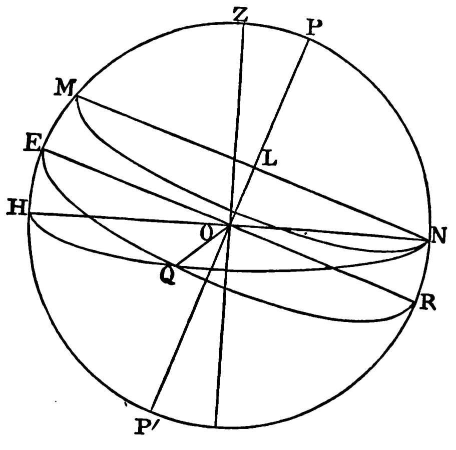
Then, as in the previous Article, the point s is by the Earth's rotation made to describe the small circle ss' uniformly, so that CS describes uniformly a right circular cone about the axis CP, in the course of a day. Now the line OS drawn from the spectator to the star is always

parallel to CS, the star being at an immense distance from the Earth. Draw OP parallel to CP: then OS describes a cone similar to the cone described by CS, and in the same direction, viz. opposite to the direction of rotation of the Earth. This cone will intersect the celestial sphere in a small circle which the star will appear to describe uniformly about the point P, the north pole of the heavens.

The stars, then, appear, during that portion of the day when they are visible, to describe portions of circles about the poles of the heavens, or parallel to the celestial equator.

105. *Time during which a star remains above the horizon depends on its declination. Appearances to a spectator at the pole, and at the equator.*

The plane of the meridian divides the celestial sphere into two hemispheres; one of these is represented in the figure, in which the plane of the meridian ZPP' is sup-



posed to coincide with the plane of the paper. Let P, P' be the poles, Z the zenith, HQN the horizon intersecting the meridian in HON, and EQR the celestial equator intersecting the horizon in OQ, which is a line perpendicular to the meridian. Then the horizon conceals from view the hemisphere $HP'N$ below it.

Consider now a star on the equator; its diurnal path is bisected by the horizon; it will therefore be visible throughout half its course, of which half the part QE on this side of the meridian is represented in the figure. Again, consider a star having north polar distance PON, and therefore moving on the circle NM parallel to the equator: its whole course is above the horizon; and it is clear that all stars with less N.P.D. describe their course entirely above the horizon. Similarly all stars with south polar distance $P'H$ or less than $P'H$ describe their whole diurnal path below the horizon. Stars with N.P.D.s greater than PN (which is equal to the latitude of the place) and less than 90^0 describe part of their course above and part below the horizon, the part above the horizon being greater than that below. Stars with S.P.D. greater than $P'H$ and less than 90^0 have more than half their diurnal circles below the horizon.

To an observer on the pole each star's diurnal circle is parallel to the horizon; for the horizon in that case is parallel to the equator. Hence all stars north of the equator are continually visible above the horizon, while stars south of the equator are always below the horizon.

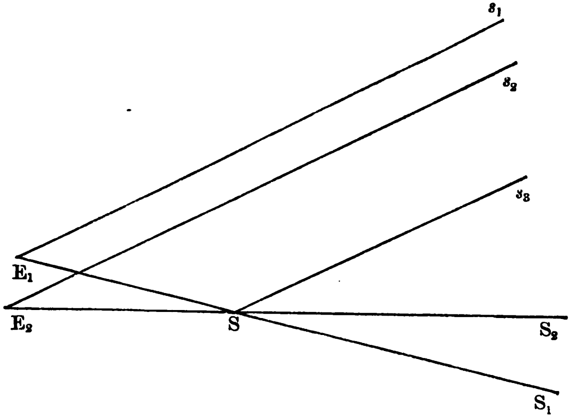
To an observer on the equator P and N coincide, as also P' and H, and the equator passes through his zenith. The diurnal paths of all stars are therefore circles perpendicular to the horizon, and each describes half its path above and half below the horizon.

106. *Apparent motion of the Sun among the stars accounted for by the orbital motion of the Earth about the Sun.*

We have next to describe the appearances presented to an observer on the Earth in consequence of its annual revolution about the Sun.

Let E_1 be the position of the Earth's centre at any time, and E_2 its position at a subsequent time. S the Sun's centre. Draw E_1s_1, E_2s_2, Ss_3 in the direction of any fixed star in the plane of the Earth's orbit. Then the angular distance of the Sun from this star as seen from E_1 is the $\angle s_1E_1S$, which is $= \angle s_3SS_1$, since E_1S subtends no measurable angle at the star, and therefore E_1s_1, Ss_3 are

parallel to each other. So the angular distance of the Sun from the star as seen from E_2 is s_3SS_2.



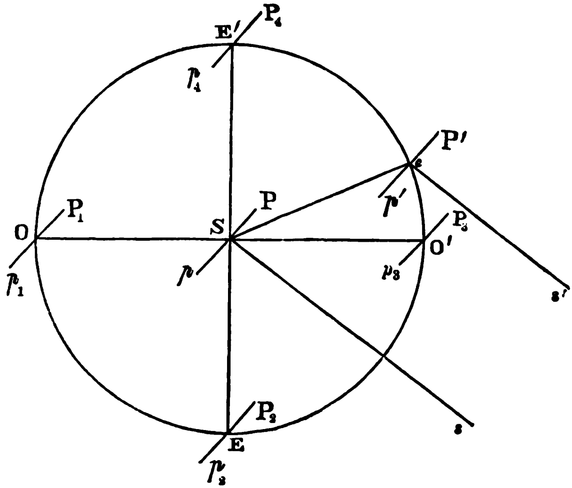
Hence the Sun will appear to have approached the star by an angle equal to S_1SS_2, which is equal to the angle E_1SE_2 described by the Earth's centre about the Sun.

Thus, in consequence of the Earth's motion about the Sun, the Sun appears to a spectator at the Earth's centre to move in the plane of the Earth's orbit, and to describe in any time an angle equal to that described by the Earth about the Sun. The angle subtended by a radius of the Earth at the Sun is very small; the angle which the Sun appears to a spectator on the Earth's surface to describe with reference to any star will therefore be almost undistinguishable from the angle as observed by a spectator at the centre. To a spectator on the Earth's surface, therefore, the Sun will appear to describe among the stars a great circle, the plane of which is that of the Earth's orbit about the Sun.

107. *Varying N.P.D. and meridian altitude of Sun accounted for by the orbital motion of the Earth.*

If the axis of rotation of the Earth were perpendicular

to its orbit, the equator and ecliptic would coincide, and the great circle which the Sun appears to describe among the stars would be the equator. Since, however, the Earth's axis is not perpendicular to the ecliptic, the Sun will appear to describe a great circle inclined to the equator, and its N.P.D. as well as its R.A. will appear to vary throughout the year.



Let S be the Sun's centre, Pp a line through it parallel to the Earth's axis; let the plane through Pp perpendicular to the plane of the Earth's orbit meet it in OO'. Draw ESE' in the plane of the orbit perpendicular to OO', and let O, E, O', E' be the positions of the Earth's centre on these lines; P_1p_1, P_2p_2, P_3p_3, P_4p_4 the positions of the Earth's axis—all parallel to Pp.

Then, since the plane EOE' is perpendicular to the plane PSO', and EE' is perpendicular to their line of intersection, EE' is perpendicular to the plane PSO', and therefore to PS; also the angles which OO' makes with Pp are the greatest and least of all the angles Pp makes with lines in the plane of the orbit.

At E and E', then, the Sun's direction as seen from the Earth is perpendicular to the axis, or the sun is in the equator; and at O and O', the N.P.D.s are the greatest and least: and since these N.P.D.s are supplementary to each other, the least S.P.D. being the supplement of the greatest N.P.D. is equal to the least N.P.D.

At O, the Sun is seen nearest to the north pole P, and as the Earth moves from O to E, the Sun recedes farther and farther from it, till at E, i.e. after an interval of a quarter of a year, the Sun is on the equator. The N.P.D. then becomes greater than 90^0, and the Sun is in south declination; the N.P.D. then increases as the Earth moves from E to O', when it is greatest, and its S.P.D is then least and equal to the least N.P.D., which it had at O. The S.P.D. then begins to increase from $p_3O'S$, its least value, to $p_4E'S$, which is 90^0; the Sun is then on the equator, and as the Earth moves from E' to O the Sun's N.P.D. diminishes from 90^0 to its least value P_1OS.

Thus in the course of a year, the Sun, after arriving at an angular distance from the north pole equal to the inclination of the Earth's axis to the ecliptic, turns back towards the equator, crosses it, and having arrived at an equal distance on its south side, approaches the equator again, again crosses it, and finally returns to its greatest distance from the equator on the north side.

The angle described in the course of a single day by the Earth about the Sun is so small that the deviation of the Sun in a day from the circle of its diurnal apparent motion is hardly noticed. The accumulated effect of the orbital motion is however very clearly evidenced by the difference in the meridian altitudes of the Sun at Midsummer and Winter.

108. *Definitions.*

The points of the Sun's orbit among the stars at which it crosses the equator are called the Equinoxes; the point through which the Sun passes into north declination being the Vernal Equinox, and the other the Autumnal Equinox.

The points of the ecliptic, 90^0 distant from the equinoxes, are called the Solstices. They are the points at which the Sun's declination is greatest, and at which therefore it turns back towards the equator.

DEF. The Solstice north of the equator is the *Summer Solstice*, and that south of it the *Winter Solstice.*

DEF. The declination-circle through the equinoxes is called the *Equinoctial Colure*, and the declination-circle through the solstices is called the *Solstitial Colure.*

The inclination of the ecliptic to the equator is $23^0.28'$. It is called the *obliquity* of the ecliptic.

DEF. Circles of latitude on the Earth's surface, distant $23^0.28'$ on either side of the equator, are called the *Tropics:* the *Tropic of Cancer* being the circle north of the equator, and the *Tropic of Capricorn* south of it.

The portion of the Earth's surface included between the tropics is called the *Torrid Zone.*

DEF. Circles of latitude on the Earth, distant $23^0.28'$ from the poles, are called the *Arctic* and *Antarctic Circles;* the Arctic circle being the circle near the north pole, and the Antarctic circle being that near the south pole.

The portions of the Earth's surface about the pole, which these circles respectively contain, are called the *Arctic* and *Antarctic Zones.*

DEF. The remaining portions of the Earth's surface, which are contained between the tropic of Cancer and the Arctic circle, and the tropic of Capricorn and the Antarctic circle, are called the north and south *Temperate Zones.*

It is easily seen that the points in which any meridian meets the Arctic circle and the tropic of Capricorn are 90^0 apart.

109. *Varying positions of the Sun throughout the year as seen by spectators in different latitudes.*

When the Sun is in the winter solstice, it is vertically over a place on the tropic of Capricorn, and is therefore on the horizon of the place in the same meridian on the Arctic circle. The same is true for the Antarctic circle and the tropic of Cancer.

A spectator *on the equator* has the Sun in his zenith at noon on the two days on which it crosses the equator. The diurnal circles of the Sun are all perpendicular to his horizon; their centres are therefore on the horizon, which bisects them all. Day and night are therefore of equal length throughout the year; and the Sun's meridian zenith-distance varies from 0^0 to $23^0.28'$ on either side.

A spectator *on one of the tropics* will have the Sun in his zenith once in the year; on the tropic of Cancer at the summer solstice, and on the tropic of Capricorn at the winter solstice.

At places *within the Torrid Zone* the Sun is in the zenith twice a year; places on the north side of the equator having the Sun at its greatest zenith-distance at the winter solstice, and places on the south side having the greatest zenith distance of the Sun at the summer solstice.

The greatest meridian zenith-distance of the Sun at a place on either of the tropics is equal to the sum of the distances of the zenith and the Sun from the equator when the Sun is most below the equator, i.e. it is twice $23^0.28'$, or $46^0.56'$. The altitude of the Sun is then equal to $90^0 - 2 \times 23^0.28'$, or $43^0.4'$.

Again, a spectator *on the Arctic Circle* has a latitude equal to $90^0 - 23^0.28'$; also the greatest and least meridian zenith-distances of the Sun are equal to the latitude + Sun's greatest angular distance from the equator, and latitude − Sun's greatest distance. Thus the meridian zenith-distance varies from 90^0 to $90^0 - 2 \times 23^0.28'$ or $43^0.4'$.

To a spectator on the Arctic Circle, therefore, the Sun once in the year—at the winter solstice—describes its diurnal circle entirely below the horizon; from the winter solstice to the summer solstice its meridian altitude continually increases, as also the length of time during the day that the Sun is above the horizon, the planes of its diurnal circles being inclined to the horizon at an angle $23^0.28'$; and at the summer solstice the Sun just does not set throughout a day.

The greatest meridian zenith-distance of the Sun at a place *in the Temperate Zone* between the Arctic Circle and the Tropic of Cancer, is intermediate between 90^0, its value at the Arctic Circle, and $2 \times 23^0.28'$, its value at the Tropic of Cancer; and its least zenith-distance is intermediate between $90^0 - 2 \times 23^0.28'$ (or $43^0.4'$) and 0^0. Thus the Sun never transits the meridian at the zenith or below the horizon; and it is always on the side of the zenith away from the north pole. Also the inclinations of its diurnal circle vary from $23^0.28'$, the value at the Arctic Circle, to $90^0 - 23^0.28'$ or $66^0.32'$, the value at the Tropic.

Again, a spectator at the pole would have the equator for his horizon; thus the greatest and least meridian zenith-distances of the Sun are $90^0+23^0.28'$, and $90^0-23^0.28'$. From the winter solstice, therefore, to the vernal equinox, and from the autumnal equinox to the winter solstice, the diurnal circles described by the Sun are all below the horizon; and the rest of the year its circles are all described above the horizon, all the circles being in planes parallel to the horizon. Thus one half of the year is night, and the other half day.

At places within the Arctic Zone the Sun during a portion of the year is entirely below the horizon, during another portion it is, part of the day, above and, part of the day, below, and during the remainder of the year it is continuously above the horizon.

The phenomena presented to an observer in the southern hemisphere will be precisely similar to the phenomena presented to an observer in the same latitude in the northern hemisphere; with the exception that the motions which appear from left to right to a person on the northern hemisphere, will appear from right to left to one on the southern, and *vice versâ*.

110. *Apparent motion of the Sun in R.A.*

The Sun, in consequence of the motion of the Earth about it, appears to move among the stars not only in declination, but also in right ascension. This we proceed to show.

In the figure on page 76, let $P'p'$ be the axis of the earth e at any point of the orbit: then Pp, $P'p'$ and Se are in the same plane; and the plane $P'eS$, which is the plane of the declination-circle through the Sun's centre, coincides with the plane PSe. If, then, Ss, es' be drawn in the direction of any star in the plane of the ecliptic, the angle between the declination-circles through the Sun and star as seen from the Earth is the angle between the planes $P'es'$ and $P'eS$, or the angle between PSs and PSe.

When e meets Ss this angle vanishes; and as e moves in its orbit round the Sun the angle increases continuously, by the revolution of the plane PSe about Pp; and the revolution of this plane about Pp will be

completed when e has come round to the position from which it started.

Thus in the course of a year the Sun separates from any fixed star in right ascension continuously in the same direction from 0^0 to 360^0.

Since the direction in which the Earth revolves about the Sun is the same as that in which it rotates about its axis, the apparent annual motion of the Sun is in the opposite direction to the diurnal motion of the firmament. Hence as the Sun describes the part of its diurnal apparent course which is above the horizon, it moves backwards by its annual motion, and is therefore longer above the horizon than if it were a fixed star. Its time of setting, and similarly its time of rising, are thus seen to be retarded by its orbital motion.

111. *Determination of the position of the line of apsides: progressive motion of the apogee.*

By Kepler's second law, the orbit of the Earth is an ellipse with the Sun in one focus. The excentricity of the ellipse is very small, being about $\frac{1}{60}$. The difference between the greatest and least distances of the Earth is therefore small, and hence the apparent size of the Sun does not vary much. If, however, accurate measures be taken of the apparent diameter of the Sun, i.e. of the angle subtended by a diameter of its disc at the observer's eye, they will be found to vary, and the greatest and least values correspond to the excentricity given above; their ratio being about 61 to 59.

DEF. The positions of the Earth at its least and greatest distances, which are at the extremities of the major axis of the orbit (called also the *line of apsides*) are called respectively the *perihelion* and *aphelion* distances: and the apparent positions of the Sun's centre are then called *perigee* and *apogee*.

Since, by Kepler's first law, the areas described by the Earth in equal times about the Sun are equal, it follows that the angular velocity of the Earth about the Sun is greatest in perihelion and least in aphelion; and if a straight line be drawn through the Sun in the plane of the

orbit perpendicular to the major axis, it will divide the ellipse into two parts, of which the part containing the aphelion is described in longer time than that containing the perihelion; and the difference is in this case considerable. The major axis divides the ellipse symmetrically, and into two halves which are described in equal times. Any intermediate line divides it into parts described in unequal times, the difference in the times being greatest in the case of the line perpendicular to the major axis, and less the less is the angle between the line and the major axis.

If then the Sun be observed in two positions in its orbit, 180^0 distant from each other, and again when it returns to its first position, the time elapsed between the first two observations will be half that between the first and last, if at the first the Sun was in apogee or perigee. If not, the time will differ from half the time between the first and last observations, and from the amount of this difference the position of the line of apsides with respect to the line joining the positions of the Sun at the first and second observations may be calculated. In this way the position of the line of apsides may be determined in any year.

By comparing determinations of its position for different years, it is found that they do not agree. The apogee and perigee do not retain invariable positions among the stars, but move among them from west to east, or in the direction of the Sun's motion: the yearly amount of this motion being 11·25″.

This is called the progressive motion of the apogee.

112. *Signs of the Zodiac.*

The ecliptic is divided into twelve equal parts, which are called the *Signs of the Zodiac.* They are called in the order of the Sun's course, Aries (♈), Taurus, Gemini, Cancer, Leo, Virgo, Libra (♎), Scorpio, Sagittarius, Capricornus, Aquarius, Pisces.

The Vernal Equinox was once in the sign Aries, but by its retrograde motion in the ecliptic has moved into Pisces. It still, however, retains the name of the first point of Aries.

The Autumnal Equinox is called also the first point of Libra, and is often designated by the sign (♎) of Libra.

113. *The Seasons.*

The four portions into which the year is divided by the successive passages of the Sun through the equinoxes and solstices are called the Seasons: the three months between the vernal equinox and the summer solstice are called Spring, the next quarter of the year Summer, the next Autumn, and the last Winter. In each of these seasons the Earth describes an angle of 90^0 about the Sun; these angles are not described in equal times, since the areas which they include are not equal. At present the position of the equinoxes with regard to the apses of the orbit is such that the order of length of the seasons is summer, spring, autumn, and winter, summer being the longest.

The variation of temperature at different seasons is due mainly to two causes, firstly, the varying proportions of day and night at different times of the year; and secondly, the varying meridian altitude of the Sun. We will treat of these two causes separately.

With regard to the first cause, it is evident that, *ceteris paribus*, the longer the Sun remains above the horizon of a place on any day the greater amount of heat will the place receive; on this account then, a place receives more than the average amount of heat from the Sun when the Sun is more than twelve hours above the horizon, and less when it is less than twelve hours. Now, the Earth is not only receiving heat continually from the Sun, but is constantly parting with heat by a process called *radiation;* and the amount of heat received from the Sun in the course of a year is about equal to the amount lost by radiation, and thus the average temperature of the year is nearly the same from year to year. In northern latitudes when the Sun passes the vernal equinox, the days become longer and the nights shorter than twelve hours, and the amount of heat then received in these latitudes exceeds the average, and is in excess of that lost by radiation; thus the temperature increases, and this increase of temperature continues not only till the Sun reaches the summer solstice, when its stay above the horizon is longest; for, during some portion of the return of the Sun towards the autum-

nal equinox, the heat received is in excess of that lost by radiation. Thus, after the Sun has passed the vernal equinox till it has arrived at some point between the summer solstice and the autumnal equinox, the temperature, so far as it depends on the cause we are considering, increases; beyond this point, as the Sun's meridian altitude diminishes, the temperature diminishes. When the Sun has reached the autumnal equinox the same variations of temperature are produced from this cause in the same order in the southern hemisphere, as the Sun passes from the autumnal equinox through the winter solstice back to the vernal equinox. Thus, in the southern hemisphere the phenomena of the seasons as regards temperature are reversed, the phenomena of the summer of the southern hemisphere happening at the time of our winter, and *vice versâ.*

The second cause which we mentioned, namely, the varying meridian altitude of the Sun, is no less important. Its influence depends on the property, that when rays from a source of radiant heat fall on a plane surface, the intensity of the heat received varies with the inclination of the rays to the surface, being greatest when they are incident perpendicularly, and less the more they are inclined to the perpendicular. Now the greater the meridian altitude of the Sun, the nearer does it approach the zenith, and the more nearly are its rays perpendicular to the Earth's surface at noon. Hence, for this cause also the temperature increases in northern latitudes after the Sun passes the vernal equinox, and in southern latitudes after it passes the autumnal.

There is still a third cause, which is not without its effect on the temperature; it is the varying distance of the Earth from the Sun, on account of the excentricity of the Earth's orbit. The greatest variation of distance amounts (Art. 111) to a thirtieth part of the whole distance. Now the intensity of the radiant heat from the Sun varies inversely as the square of the distance; hence, the direct heating effect of the Sun's rays is greatest in perigee, and least in apogee, the difference amounting to a fifteenth part of the whole. Now, at the present time, the position of apogee is (Art. 126) near the summer solstice; thus the

intensity of the Sun's rays is, on this account, least in summer and greatest in winter, and the effect of the excentricity is to mitigate the heat of summer and the cold in winter in the northern hemisphere, and so to equalize the seasons in regard to temperature: in the southern hemisphere the effect is the reverse, and the difference of temperature in different seasons is increased.

Though the effect is thus to make the range of temperature less in the northern hemisphere and greater in the southern, the whole amount of heat received in any one of the seasons is not affected by this cause. For, from Kepler's first law, it follows that the angular velocity of the Earth about the Sun, that is, the rate at which angles are described by the line joining the centres of the Earth and Sun, is greater the less the distance, being greatest at perigee when the Sun's direct heat is greatest, and least at apogee; and it can be shewn that this angular velocity varies inversely as the square of the distance, and therefore directly as the intensity of the Sun's heat; and thus, the heat received in any time varies as the angle described by the Earth about the Sun in that time, and equal amounts of heat are received while the Earth describes equal angles. Hence, the total amount of heat received is the same in each season; and again, the amount of heat received by a place in either hemisphere is the same during the same time when the Sun is north as it is when he is south of the equator.

114. *Difference of weight of a body in different latitudes.*

Using the word weight in the sense of the pressure which a body at rest on the Earth's surface exerts on another by contact with which it is supported, we proceed to shew that the weight of a body depends on the latitude in which it is weighed.

A body at either of the poles is kept at rest by two forces—the attraction of the Earth and the supporting pressure. The pressure is therefore equal in magnitude and opposite in direction to the force of gravity at that point.

This is nearly but not accurately true at other points of

the Earth's surface; for in consequence of the rotation of the Earth, a body at relative rest on its surface, at all places except the poles, is not really at rest, but describes a circle about a point on the Earth's axis as centre. This circle requires (Newton, Section II., Prop. 14) a force tending to the centre of the circle; hence the pressure is not exactly equal and opposite to the force of gravity, but is inclined to it at a small angle, and the two forces have a resultant tending to the centre of the circle described by the body.

At a place on the equator the centre of the circle described is the centre of the Earth; at such a place, therefore, the direction of the pressure must be opposite to that of the Earth's attraction, but not equal to it; the attraction must be sufficiently in excess of the pressure to maintain the body in its circular motion.

The weight of a body at the equator, as measured by the pressure which it exerts, or which is required to support it, is, therefore, less than at the poles in consequence of the Earth's rotation: it is also less on account of the ellipticity of the Earth (Art. 3), for the force of gravity varies inversely as the square of the distance, and is, consequently, less at the equator than at the pole. The excess of pressure at the pole over that at the equator from the combination of both these causes is about a 194th of the pressure at the equator.

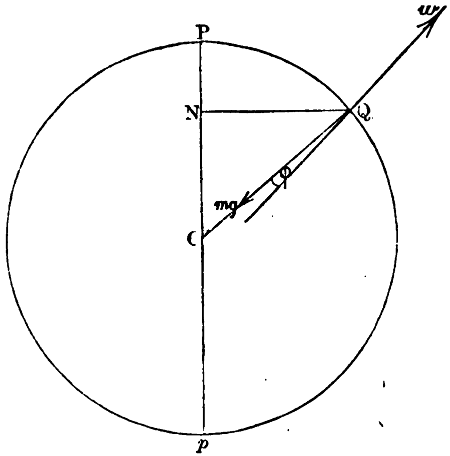
115. *Determination of loss of weight in any latitude due to the rotation of the Earth.*

We will now find an expression for the loss of weight in any latitude due to the Earth's rotation.

Let Q be the position of the body: Pp the axis, C the centre of the Earth. Draw QN perpendicular to Pp: join QC. Let Qw be the direction of the pressure which supports the weight of the body: this direction nearly but not quite coincides with QC; let wQ produced make a small angle ϕ with QC.

Let w be the weight of the body, m its mass, mg the attraction of the Earth upon it: then w and mg have a resultant in the direction of QN, the radius of the diurnal circle which Q describes about Pp. Let $QN=r$; and let v be the velocity with which Q describes its circle about

N, and ω the angular velocity of the Earth; then $v = \omega r$;



and the force to the centre by which the circle is described

$$= m\frac{v^2}{r} = m\omega^2 r.$$

Hence, w in direction Qw, and mg in direction QC, have a resultant $m\omega^2 r$ in direction QN.

Resolve these forces and their resultant parallel and perpendicular to QC: then

$$\begin{aligned} mg - w\cos\phi &= m\omega^2 r \sin PCQ \\ &= m\omega^2 a \sin^2 PCQ, \end{aligned}$$

where a is the radius of the Earth.

Now, since ϕ is very small, $\cos\phi = 1$ approximately; also $\angle PCQ = 90^0 - \lambda$, where λ is the latitude of the place; thus

$$mg - w = m\omega^2 a \cos^2\lambda;$$

or, *the loss of weight caused by the rotation varies as the square of the cosine of the latitude of the place;* in other words, it varies as the square of QN the radius of the diurnal circle.

If a body be weighed in different latitudes in a balance, since the pressures of the body and of the counterpoise are

affected equally by a change of latitude, their differences in different latitudes produce no effect on the equilibrium; by this means, therefore, we cannot discern *differences* in the pressure of a body in different latitudes.

If, however, the pressure of the body be employed to stretch a spring, the spring will be stretched to a greater length the higher the latitude, and to its greatest length at the poles, and least at the equator.

Again, the number of beats which a pendulum of given length makes in a given time is known to vary as the square root of the force under which the pendulum moves. At the equator therefore the number of beats would be less than at the pole.

By experiments on the stretching of a spring and the beats of a pendulum the result above given for the difference of the pressures exerted by a body at the pole and equator has been obtained.

116. *Trade Winds.*

It has been stated that the Sun attains an angular distance from the equator of 23°.28′ on either side, which it never exceeds. The sun is consequently always vertical at some point or other within the tropics, and never vertical at any place beyond them. Now, the heating effect of the Sun's rays at a given instant varies with the inclination to the surface on which they fall. Thus when the Sun is vertical the rays have much more effect in heating the surface of the Earth than when it is on or near the horizon: and the effect varies as the cosine of the angle of incidence, i.e. of the angle which the rays make with the normal to the surface on which they are incident. Within the tropics therefore, at some point of which the Sun is always vertical, a greater amount of heat is accumulated than at other parts of the Earth: and this heat is communicated to the superincumbent air. By this heating of the air about the equator, in conjunction with the rotation of the Earth, the phenomena of the Trade Winds are produced, as we proceed to shew.

The heated air expands, and thus becomes specifically lighter: it therefore ascends, and a partial vacuum is produced, which is immediately filled by air which rushes in

from parts nearer the poles; the heated air which has ascended, being unopposed, spreads in both directions towards the poles: it is cooled in its passage, and descends on parts of the Earth beyond the tropics. Thus far, then, the effect would be a north and south wind moving toward the equator. This effect, however, is modified by the rotation of the Earth in a remarkable manner. The velocity of a point on the Earth, in consequence of its rotation, is proportional to the radius of the circle of latitude. The cold air which supplies the place of the heated air about the equator has, at starting for the equator, a velocity less than that of the air within the tropics. By the friction of the Earth's surface, as it gets into larger circles of latitude, its velocity is to a certain extent increased, but there remains still some deficiency of the velocity as it arrives at places within the tropics. It therefore lags behind the Earth, and as the Earth rotates from west to east, the air has on this account a motion relatively to the Earth's motion, from east to west. This motion, combined with the general direction of the wind from the poles to the equator, produces the effect of a permanent north-easterly wind at places within the tropics in north latitude, and south-easterly in south latitude. Near the equator the circles of latitude vary very slightly: the easterly tendency therefore of the wind is slight, so that, by the time the air arrives at the equator, it has been destroyed by the friction; and the northern and southern currents meet and counteract each other. Thus the Trade Winds entirely disappear at the equator.

117. *Tides.*

It remains to mention another remarkable effect of the rotation of the Earth in conjunction with the attractions of the Moon and Sun, which is the phenomenon of the *Tides.* These are produced by the joint attractions of the Moon and Sun on the Earth and the waters which surround it. The Moon produces the principal effect on account of its greater proximity, the effect of the Moon being greater than that of the Sun in the proportion of about 5 to 2.

Since, by the law of gravitation, the waters of the Earth which are immediately under the Moon are attracted with a greater accelerating force than the Earth itself, they will

be drawn away from the Earth with a force equal to the difference of the attractions. So, the waters on the opposite side of the Earth will be attracted with less accelerating force than the Earth: the land is therefore drawn away from the waters on this side, and the result is the same as if the waters were drawn away from the land. The waters, then, are drawn away from the land at both extremities of a diameter of the Earth drawn in the direction of the Moon.

It has been proved that the shape assumed by the surface of the waters by the attraction of the Moon must be a prolate spheroid; this supposes the surface of the Earth to be entirely covered by water. In the actual case, the spheroid above mentioned could not be formed entire, but parts of a spheroid would be formed interrupted in various parts by the land.

We have not at present taken account of the rotation of the Earth. In consequence of the rotation, the vertices of the spheroid are relatively to the Earth in continual motion in a direction contrary to the rotation of the Earth, or from east to west, so as to follow the apparent diurnal motion of the Moon.

When the water which is at the highest level strikes the coast at any place, it is *high water;* and when the water which is at the lowest level strikes it, it is *low water.*

In the above account we have for simplicity neglected the action of the Sun, which produces precisely a similar, though a less effect; and it is by the combined action of the two bodies that the actual wave is produced which results in the tides.

When the highest levels produced by the Sun and Moon are in the same direction from the centre of the Earth, which happens when the Moon is near conjunction and opposition, i. e. when its angular distance from the Sun is nearly 0^{0} and 180^{0}, the resulting high tide is at its highest possible, and the low tide at its lowest possible: the tide is then called a *spring tide,* and the difference between high and low tide is the greatest possible. When the Moon is near quadratures, i. e. when its angular distance from the Sun is about 90^{0}, the solar and lunar waves tend mutually to destroy each other, although the lunar wave predominates; the tide is then called a *neap tide.*

The investigation of the shape assumed by the surface of the waters in consequence of the attractions of the Moon (or of the Moon and Sun), and of the Earth and water, of the fluid pressures, of friction, and of rotation, is a complex hydrodynamical problem which does not admit of non-mathematical explanation. The proposition that, in the case of *a shallow equatoreal canal*, the low tides due to the Moon's (or Sun's) attractions and to rotation are in the line joining the centres of the Earth and the attracting body was, however, proved by Newton in a simple manner in the *Principia* (Lib. I., Prop. 66, Cor. 19). For an elegant demonstration of this proposition by Sir G. B. Airy, the reader is referred to the *Monthly Notices of the Royal Astronomical Society for the year* 1866.

This result agrees with Laplace's more complete investigation of the tides applied to a similar case.

The elevations and depressions due to the tidal actions of the Moon and Sun had been calculated by Newton in the *Principia*, and the whole theory of the tides was afterwards investigated by Daniel Bernoulli as a case of equilibrium; the form of the waters on this theory is supposed to be not widely different from what it would be if the Earth and waters had no rotation; and the flow and ebb of the tides at any place are supposed to be similar to what they would be if the solid Earth rotated under the waters, the surface of which assumed the shape given by this supposition.

This has been called the 'Equilibrium Theory'; and if this theory were applied not only to giving the shape of the waters but the positions of the tidal phases relatively to the Moon, the high tide due to the Moon would be directly under the Moon.

Both Laplace's and the Equilibrium Theory give an account of the state of the tide *in the open sea* on certain suppositions, and in an imaginary case of distribution of land and water. The actual time of high water at any place is greatly affected by physical obstacles to the progress of the tide-wave, such as the length and breadth of the channels through which it has to pass.

The time of high water at any place, on the days of new and full Moon, is called the *Establishment of the Port:* for places on the west coast of Ireland most exposed to the Atlantic the time is about 4 o'clock or a few minutes earlier.

If the Moon were the only attracting body, the time of high tide would differ from the time of the Moon's transit on any day, only by the Establishment of the Port. The Sun, however, being sometimes before and sometimes behind the Moon, the time of high tide, being due to the combined effects of the two bodies, will differ on different days; and the interval between successive high tides will vary. The tidal day will therefore differ from the lunar day or the interval between successive transits of the Moon.

This difference is called the *priming* and *lagging* of the tides.

118. *Proofs of the Earth's rotation.*

In the explanations in this chapter we have assumed that the Earth rotates about an axis through its centre and revolves about the Sun. It is possible, however, to explain some of the phenomena by supposing the Earth to be fixed, and the heavenly bodies to revolve about it once a day: we will, therefore, conclude this chapter by giving certain reasons for believing that the Earth really does rotate about an axis and revolve about the Sun.

That the Earth rotates about an axis is proved by the following considerations:

(1) It not only explains the fact of the apparent motions of the heavenly bodies about the Earth, but accounts for the apparent paths of the stars being all in parallel planes, and all described in the same time. If the Earth do not rotate we can only explain this by supposing the heavenly bodies to be parts of one rigid body; but this supposition is inconsistent with the motions which the Sun, Moon and planets appear to have among the fixed stars.

(2) By observations taken at different points of the Earth's surface the distance of the Sun has been deter-

mined and is known with tolerable accuracy. Its distance being known, and its apparent diameter, it is easy to calculate its volume: now the distance is no less than 92 millions of miles, whence it can be shewn that its volume is about a million times that of the Earth. It seems, therefore, much more reasonable to suppose that the Earth is rotating once a day, than that such an enormous body as the Sun revolves about the Earth, a comparatively minute body, with such enormous rapidity as to describe in one day a circle having a radius of 92 millions of miles.

Again, the stars are at distances vastly greater than that of the Sun; for by observations at distant parts of the Earth it is known that a diameter of the Earth does not subtend the smallest appreciable angle at any one of them; they would require, therefore, a velocity very much greater even than that required for the Sun.

(3) Certain spots on the surface of the Sun have been observed to move across the disc from the eastern to the western side of it, and then disappear, the time occupied in crossing the disc being rather less than a fortnight; after another interval of nearly a fortnight the spots have been observed to reappear and cross the disc again in the same time, and so on several times over. This can only be explained by supposing the Sun to rotate about an axis, the period of rotation being about 25 days.

Similar observations of well-defined marks on the surfaces of those planets which come sufficiently near to the Earth to permit of such observations have led to the conclusion that they rotate; thus Venus, whose size is very nearly equal to that of the Earth, rotates in a period of $23\frac{1}{4}$ hours. The analogy of the Sun and planets is, therefore, in favour of the supposition that the Earth rotates.

(4) The figure of the Earth is an oblate spheroid, which is precisely the form which it would have taken if it had been at one time fluid, and had a rotation about an axis through its centre; and the diameter about which it rotates would be the shortest; thus rotation about the polar diameter would account for the Earth deviating from sphericity, and having the form which it actually has.

(5) Experiments with pendulums at different points on the Earth's surface have proved that the variation of weight in different latitudes agrees with calculations based on the supposition that the Earth rotates about its polar axis.

(6) If the Earth rotates about its axis, points on the Earth will move faster the farther they are from the axis, for they will describe larger circles: hence the top of a tower moves eastward in consequence of the rotation faster than the foot of it. If, therefore, a ball be dropped from the top of a tower, it will have a vertical motion on account of the action of gravity, and in addition the horizontal motion which it had at the top of the tower on account of the Earth's rotation; this horizontal velocity being greater than that of the foot of the tower, it will by the time it reaches the ground have moved farther eastward, and will therefore fall a little to the east of the foot. Experiment has demonstrated the existence of such a deviation when a ball has been dropped from the top of a tower, and from the top of the shaft of a mine.

(7) It is found from mechanical considerations, that the effect of the rotation of the Earth on the plane of oscillation of a pendulum freely suspended will be to separate the plane of the meridian from this plane at a rate which depends on the latitude of the place; and given the latitude, the rate of separation and the angle between the planes after a given time can be calculated. Observations made with a pendulum by M. Foucault in Paris and by other observers in various places have confirmed the calculated results.

(8) To M. Foucault is due another experiment for the purpose of demonstrating the rotation of the Earth. It follows from mechanical principles that, if a body which is symmetrical about a certain line through its centre of gravity be made to rotate about this line as axis, and if it be supported in such a manner that the supporting forces are equivalent to a force through the centre of gravity, and that its axis can move freely in any direction, neither the supporting forces nor the force of gravity will affect either the angular velocity of rotation or the position of the axis of rotation in space. M. Foucault supported a heavy

revolving disc under the conditions we have mentioned, and found that the axis had an apparent motion such as it would have if the Earth were rotating while the axis of the disc retained a fixed direction in space. The revolving disc used in these experiments is known as the *gyroscope.*

119. *Proofs of the Earth's annual motion round the Sun.*

From mere observations of the apparent motion of the Sun among the stars, it is impossible to infer whether the Earth has an orbit round the Sun, or the Sun round the Earth, for the phenomena would be the same on either hypothesis. The real proof of the annual motion of the Earth rests on the very accurate way in which the apparent motions of the planets are explained on this supposition, and the rigorous agreement between the results of theoretical Astronomy, and of observations.

The following considerations may be mentioned as so many special grounds for coming to the same conclusion.

(1) The great size of the Sun compared to that of the Earth renders it easier to suppose the Earth to revolve round the Sun than the contrary.

(2) If we assume the truth of the law of gravitation, we are left in no doubt about the matter. For, by Newton, Section III. Prop. XV., $P = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}}$, where P is the periodic time in which a body describes an ellipse round a centre of force in the focus, a the semiaxis major of the ellipse, or the mean distance, and μ the acceleration produced by the force at distance unity, or, as it is called, the *absolute force.*

Let, now, S, E, M be the absolute forces of the Sun, Earth, and Moon; then $\frac{S}{r^2} + \frac{E}{r^2}$ is the sum of the accelerations with which the Earth is urged to the Sun, and the Sun to the Earth, by their mutual attractions at distance r; thus $\frac{S+E}{r^2}$ is the acceleration of the Earth relative to the Sun, that is, it is the acceleration which the Earth would have if the Sun were fixed and the Earth moved

in the same manner relatively to the Sun as it actually does: hence the absolute force in such relative orbit would be $S+E$. Similarly, the absolute force for the relative orbit of the Earth and Moon would be $E+M$.

Now, the periodic times in the apparent orbits of the Sun and Moon are known, and their mean distances: hence, by the relation above given, the ratio of $S+E$ to $E+M$ is known; this ratio is about 355,000 to 1. Hence $E+M$, and, *à fortiori*, E, is extremely small compared with $S+E$; thus E is very small compared with S; now the absolute forces are proportional to the masses; hence the mass of the Earth is extremely small compared with that of the Sun, and therefore the centre of gravity of the Sun and Earth is near the centre of the Sun. Again, the centre of gravity of two bodies which are only acted upon by their mutual attractions either is at rest or moves uniformly in a straight line. Neglecting, therefore, the small disturbances produced by the other bodies of the solar system, we may say, speaking roughly, that the Sun is either at rest or moving uniformly in a straight line, while the Earth describes annually an ellipse about it.

(3) By Kepler's third law the planets describe ellipses about the Sun in periods whose squares are as the cubes of their mean distances from the Sun; now, the relation between the mean distance of the Earth from the Sun and those of the planets, is found to be just what it should be according to this law, if the Earth is a planet, and its period a year. We therefore have reason to infer that the Earth is a planet and revolves round the Sun, as the other planets do.

(4) The assumption that the Earth moves round the Sun, combined with the assumption that light moves with immense though finite velocity, enables us to explain certain observed apparent displacements of the stars, and to account accurately for them.

(5) The annual motion of the Earth affords the simplest explanation of the stationary points and retrograde motions of the superior planets.

CHAPTER V.

ON TIME.

120. *Mean Solar Day.*

Our ideas of time are derived from the succession of events or phenomena; and the most convenient phenomena to which we can all refer are supplied by the motions of the heavenly bodies.

By means of these we get our popular ideas of a day, a month, and a year; thus from the alternations of light and darkness caused by the rising and setting of the Sun we derive our idea of a day. For astronomical purposes, however, we require a fixed epoch by which the beginning and the end of the day are unmistakeably marked, in order that we may have a starting-point from which any event in the day may be measured. This is furnished for any given place by the transit of the Sun's centre over the meridian of the place; at the moment at which this happens, one day ends and the next begins.

The interval between two successive transits of the Sun over the meridian is called a *solar day*.

The instant of its passage is called *noon*.

It is necessary to divide the day into smaller intervals. This is effected by means of clocks, the hands of which rotate uniformly a certain number of times in the course of the day. If a clock could be regulated so that the hands

should always come to a given position at noon, the position of the hands of the clock would always indicate the time from noon. Since, however, the interval of time between successive transits of the Sun varies slightly from day to day, this cannot be done.

Though the solar day is not actually of constant length, it has a *mean* or average length, than which it is never much shorter or much longer. This mean day may be conceived to be determined by the successive transits of an imaginary Sun—called the mean Sun—over the meridian, starting with the real Sun at some assumed epoch and crossing the meridian at equal intervals of time. The interval between successive transits of this imaginary Sun is called a *mean solar day.*

A clock regulated to complete its period in a mean solar day is called a mean solar clock.

121. *Sidereal Day.*

Astronomers use besides the mean solar day another measure of time; it is the interval between successive transits of the first point of Aries over the meridian of the place of observation, and begins at the instant of the transit, and is called a sidereal day: if the first point of Aries were a point absolutely fixed among the stars, this interval would be exactly equal to the interval between successive transits of any fixed star, or to the period of rotation of the Earth. The motion of the first point of Aries is, however, so slow, being only about $50''$ yearly, that no practical inconvenience is felt in assuming the sidereal day to be absolutely constant and to be equal to the period of rotation of the Earth.

A clock regulated so as to complete its period in a sidereal day is called a sidereal clock.

The mean solar and sidereal days are both divided into twenty-four equal intervals called hours, each hour into sixty minutes, and each minute into sixty seconds.

A mean solar clock should indicate $0^h, 0^m, 0^s$, at the instant of mean noon, or of the passage of the mean Sun across the meridian; thus the astronomical mean solar day begins at the instant of noon of the civil day; a sidereal clock should indicate $0^h, 0^m, 0^s$ at the instant of the passage of the first point of Aries across the meridian.

122. *Motion of the Mean Sun.*

We will now proceed to shew that the mean solar day is the interval between successive transits of an imaginary Sun, which describes the equator with the Sun's mean motion in longitude.

The lengths of the solar days are unequal for two reasons: (1) the unequal (apparent) motion of the Sun in the ecliptic, the motion being greatest when the Sun is in perigee and least when in apogee : (2) the non-coincidence of the ecliptic and the equator. For if the ecliptic coincided with the equator, and the Sun moved equably in it, the days would be of equal length; thus, if P = the period of rotation of the Earth expressed in terms of any unit of time, p the period of a complete revolution of the mean Sun in the equator, the angular separation of the meridian at any place from the Sun after an interval t from its transit = angle described by the meridian *minus* that described by the Sun in time t,

$$= 2\pi \frac{t}{P} - 2\pi \frac{t}{p} :$$

and, when this is $= 2\pi$, the Sun will transit the meridian again: the time between successive transits is therefore given by the equation

$$\frac{1}{P} - \frac{1}{p} = \frac{1}{t},$$

which gives t constant; hence, the length of the day, as determined by successive transits of this imaginary Sun, is constant.

The day, as determined by transits of the mean Sun, must not only be of *constant* length but must be of the same average length as the true Solar day. The mean Sun must therefore describe the equator with the Sun's *mean* motion in longitude.

Conceive now an imaginary star to start with the Sun at perigee and to describe the ecliptic with the Sun's mean motion; the Sun and star will be always together at apogee and perigee. Again, suppose at the instant this star crosses the equator at the vernal equinox an imaginary Sun, describing the equator with the Sun's mean motion in longitude, to be at the same equinox; it will describe the

equator at the same rate that the star describes the ecliptic, and the star and imaginary Sun will be together at the equinoxes. The real Sun in the ecliptic is never far from this imaginary Sun in the equator, and moves, sometimes faster, sometimes slower, and the two return to the same positions at the completion of each revolution of the true Sun in its orbit.

123. *Equation of Time.*

The noon and length of the mean solar day are determined by the transits of the imaginary Sun in the equator, which is called the *mean* Sun.

The meridian of any place separates uniformly from the mean Sun by an angular interval of 360° in twenty-four mean solar hours; hence, in t mean solar hours the meridian will separate by an angle A°, where $A^\circ : 360^\circ = t^h : 24^h$. Thus in a mean solar hour, the meridian separates from the mean Sun by an angle of 15°. When the angle contained by any two meridians, or, which is the same thing, the angle subtended by the arc of the equator which they intercept, is represented in hours by dividing the number of degrees in it by 15, the angle is said to be *reduced to time.*

Def. If the angle between the meridian through the true and the mean Sun be reduced to time the result is called the *equation of time.*

124. *Equation of time due to the Sun's unequal motion in the ecliptic, and to the obliquity.*

From the definitions of the equation of time and of the mean solar day it follows that, at the moment when the true Sun transits the meridian, the equation of time is the time in mean solar hours which elapses between the transits of the true and mean Sun.

The equation of time is considered *additive* when the *mean Sun transits first;* i. e. when it is to be added to apparent time to get mean time; and *subtractive* when it is to be subtracted from apparent time to get mean time.

The interval of time by which the mean Sun precedes the true is the sum of the intervals by which the mean Sun precedes the imaginary star, and the star precedes the true Sun. Should they follow each other in a different order, the above statement will still be correct, if the

reverse order be indicated by a negative sign. Thus, if the star precedes the mean Sun, the interval between them must be subtracted instead of being added, and the result, if negative, would indicate that the true Sun transits before the mean.

The interval by which the fictitious star in the ecliptic precedes the true Sun would be always zero, if the Sun moved *uniformly* in the ecliptic: it is, hence, called the equation of time *due to the Sun's unequal motion in the ecliptic;* and the interval by which the mean Sun precedes the star, which would be always zero if the ecliptic coincided with the equator, or if there were no obliquity, is called the equation of time *due to the obliquity.*

Thus, the equation of time is the algebraical sum of the parts of it due to the unequal motion and to the obliquity.

The equation of time is given in the Nautical Almanac for mean, and apparent, noon of each day in the year. Its greatest value is rather more than 16 minutes.

125. *The equation of time vanishes four times a year.*

We now proceed to explain the variations of the equation of time in the course of a year. In order to do this we must consider

(1) The interval of time between the transits of the true Sun and the fictitious star in the ecliptic, and

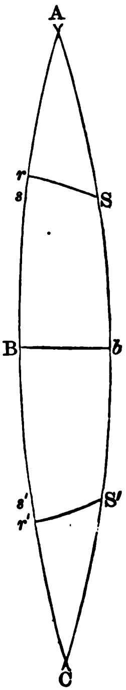
(2) The interval between the transits of the star and the mean Sun.

(1) Since, by Kepler's first law, the Earth describes about the Sun equal areas in equal times, its angular velocity about the Sun will be greatest at its least distance, and least at its greatest: in other words, the apparent orbital motion of the Sun will be greatest in perigee and least in apogee; its mean value will therefore be less than that in perigee and greater than that in apogee.

The true Sun will therefore be ahead of the star from perigee to apogee, and they will be together again in apogee. Hence, any meridian of the Earth will, by the diurnal motion of the Earth from west to east—which is also the direction of the Sun's orbital motion—be made to pass through the star before the Sun; or the star will

transit before the true Sun from perigee to apogee. So it may be shewn that the star will transit after the true Sun from apogee to perigee. Hence, the equation of time due to the Sun's unequal motion is additive from perigee to apogee, and subtractive from apogee to perigee, and vanishes at perigee and apogee.

(2) Let S, s be simultaneous positions of the fictitious star in the ecliptic and the mean Sun in the equator, when the star is between the vernal equinox A and the summer solstice B; S', s' simultaneous positions of them when the star is between the summer solstice and the autumnal equinox C. Take Ab on the ecliptic $= AB = 90^0$ so that A is the pole of bB, and bB is perpendicular to the equator. Draw the arcs Sr, $S'r'$ perpendicular to the equator: then, in the right-angled triangle ASr, the hypothenuse AS, which is equal to As, is greater than the side Ar. Hence any meridian of the Earth in its diurnal motion from west to east, which is in the direction of the Sun's orbital motion, will pass through Sr before s; thus the mean Sun transits *after* the star; when the star and mean Sun arrive at b and B they are on the meridian together.



Again, CS' is greater than Cr', and is equal to Cs'; hence, the meridian will pass through s' before $S'r'$; or the mean Sun will transit *before* the star. Thus, between the vernal equinox and summer solstice the equation of time due to the obliquity is subtractive, and additive between the summer solstice and autumnal equinox. By reasoning in the same way with regard to the other half of the Sun's orbit, we shall find that when the Sun is moving from *either* equinox to the succeeding solstice, the *star transits* first, or the equation of time due to the *obliquity is* subtractive; and when it is moving from either

solstice to the succeeding equinox, the equation of time due to the obliquity is additive.

If the mean Sun transits first, the fictitious star next, and the true Sun last, the equation of time and its component parts are all additive. Thus, when the Sun is between perigee *p* and apogee *a*, fig. (I.), the interval between the Sun and star, or the equation of time due to the Sun's unequal motion, is positive: between *a* and *p* it is negative.

Fig. I.

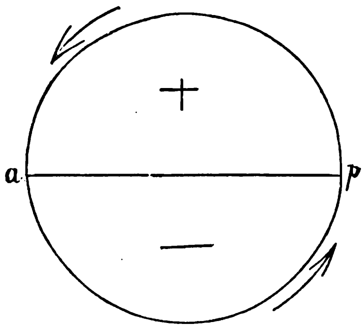
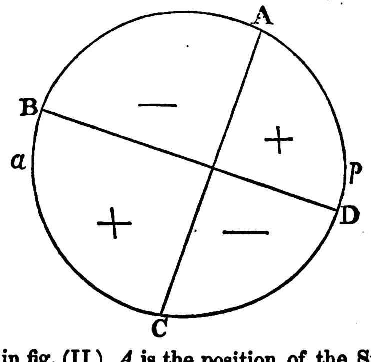


Fig. II.

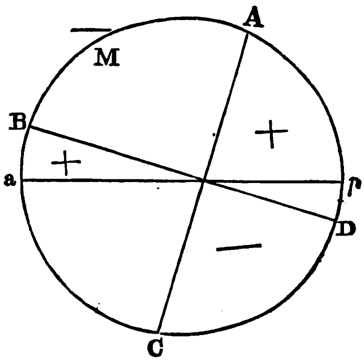


Again, in fig. (II.), *A* is the position of the Sun at the vernal equinox; *B* at the summer solstice, which at the

present time *just precedes* apogee: C is the autumnal equinox and D the winter solstice.

The interval between the star and the mean Sun is positive from D to A and from B to C, and negative from A to B and from C to D. Thus, by comparing figs. (I.) and (II.), we see that the whole equation of time is positive from B to a and from p to A, and *negative* from C to D: as represented in fig. (III.): it must therefore vanish

Fig. III.



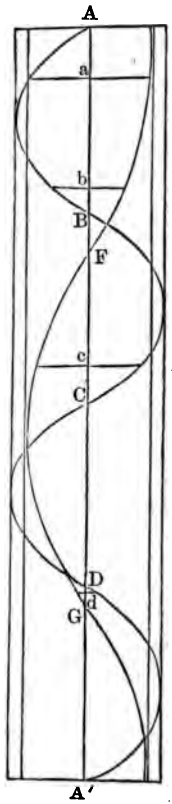
when the Sun is somewhere between a and C and between D and p; again, since the interval between the star and mean Sun vanishes at A and B, at some point M between A and B, this interval, which is the equation of time due to the obliquity and is negative, is a maximum; and this maximum is known to be greater than the maximum equation of time due to the unequal motion; hence at M the whole equation of time is *negative:* and it is positive at A and B; therefore it vanishes once between A and M, and once between M and B.

Thus, altogether, the equation of time vanishes four times in the course of a year.

126. *Facts to be remembered.*

The student is recommended to make himself thoroughly familiar with the following facts, in remembering which *consists the chief* difficulty of the foregoing proposition.

Firstly, the equation of time is considered *additive* when the mean Sun transits first; or, in the case of the equation due to the unequal motion only, when the fictitious star in the ecliptic transits first. In other words, the equation of time is *added* to the apparent time to get the mean, when it is additive, and subtracted when it is subtractive.



Secondly, the summer solstice occurs shortly before apogee;—the Sun is in the summer solstice on the 21st of June, and in apogee on the 29th of June.

Thirdly, the equation of time due to the obliquity is greater than that due to the unequal motion.

127. *Variation of equation of time during a year represented graphically.*

The variation in the equation of time throughout the year may be represented graphically by taking a line AA' to represent the time elapsed between successive arrivals of the Sun at the vernal equinox: the time is divided into four parts by the points B, C, D, which represent respectively the instants of the summer solstice, autumnal equinox, and winter solstice.

Lines are drawn perpendicular to AA', representing in magnitude the equation of time due to the obliquity, and drawn to the right of AA' when the equation is additive, and to the left when subtractive. Their extremities form the curve through A, B, C, D, A'.

Again, lines are drawn in the same manner representing on the same scale the equation of time due to the unequal motion. Their extremities form a curve passing

through F and G which mark the instants of apogee and perigee.

The actual equation of time at any instant is found by adding the lines perpendicular to AA', through the point on AA' to which the time corresponds, terminated by the curves; each line being taken with its proper sign.

It is easily seen that at two times between A and B, marked by a and b, the parts of the equation of time are equal and of opposite sign; at these instants therefore the equation of time vanishes; it will also be seen that it vanishes once between F and C, and once between D and G; thus it vanishes four times in the year.

The days on which the equation of time vanishes are April 15, June 15, August 31, and December 24.

When the position of the true Sun with respect to perigee is given, the position of the imaginary star which started with the Sun at perigee, is known with reference to the position of perigee. And the position of the imaginary star being known, that of the imaginary Sun, which starts with the star at the vernal equinox, is known with reference to the position of the vernal equinox.

Thus, if the position of perigee were fixed with reference to the vernal equinox, the real Sun and the imaginary Sun would both return to precisely the same positions with respect to the equinox after a complete revolution.

Speaking accurately, neither the perigee nor the vernal equinox is fixed; but perigee has a progressive motion, i.e. a motion in the direction of the Sun's orbital motion, in the plane of the ecliptic, of about 11″ yearly, and the vernal equinox has a retrograde motion of about 50″. By the combination of these motions, therefore, perigee approaches the vernal equinox, in the direction of the Sun's motion, at the rate of about 61″ yearly.

This very minute motion can perceptibly affect the positions *inter se* of the real and imaginary Sun and the vernal equinox only after the lapse of a great number of years.

128. *Mean tropical year.*

The apparent course of the Sun's centre among the stars is a great circle called the ecliptic, meeting the

equator in two points. The point which the Sun's centre occupies when its declination changes from south to north is (Chapter I. Art. 25) called the first point of Aries. This point, as has been explained, is not fixed on the equator, nor does it coincide with a point on the ecliptic at a fixed distance from perigee. Hence, the interval between successive arrivals of the Sun at this point is different, both from the interval between its successive arrivals at a declination-circle passing through a fixed point, and also from the interval between its successive arrivals at perigee.

The interval of time between successive arrivals of the Sun's centre at the first point of Aries is called a *tropical year*. It is, in fact, the time in which the Sun moves in R.A. through 360^0.

The tropical year is not accurately of constant length on account of the motion of the first point of Aries not being quite uniform. Its variations are, however, minute and periodic: it will therefore have a mean value. This mean value is found by comparing with recent observations of the Sun observations made many years back. His motion in R.A. in the interval between the observations may thus be found. Hence, assuming his motion in R.A. to be uniform, the interval of time corresponding to a change in R.A. of 360^0 can be found by a simple proportion.

In this way the length of a mean tropical year is found to be 365·242218 mean solar days.

129. *The Calendar.*

It is impossible practically to employ a fractional number of days in a year. The nearest integral number of days is 365: if this were adopted as the length of the year, we should make an error of nearly a quarter of a day each year, so that in four years the vernal equinox would fall one day later, and in a century twenty-five days or nearly a month. By the accumulation of this error, therefore, the seasons would in the course of time recur at quite different times of the year.

It was in consequence of this inconvenience, that, about 45 B.C., Julius Cæsar made a year to consist of 365 days, but intercalated a day every fourth year, in February. This

alteration of the Calendar is called the Julian correction. The year on which the day is intercalated is called leap-year.

The Julian correction supposes the year to consist of 365 days and a quarter; or of 365·25 days. This is too great by ·007782 of a day; and this fraction is equal to $\frac{3 \cdot 1128}{400}$ of a day. Thus in 400 years the vernal equinox would fall rather more than three days earlier.

In order to correct, and prevent the recurrence of, this error, Pope Gregory XIII. in the year 1582 caused ten days (the amount of the error accumulated since the council of Nice in 325) to be omitted after the 4th of October, so that next year the vernal equinox occurred on the same day as it had done in the year 325, namely, March 21; and he caused three days to be omitted every 400 years, by ordering that the leap-year which occurred at the completion of every century not divisible by 400 should be henceforth a common year. Thus, 1700, 1800, 1900 are common years: but 2000 is a leap-year.

This correction—called the Gregorian correction—was adopted in England in the year 1752.

130. *Sidereal day; sidereal year; anomalistic year.*

Since the mean Sun describes the equator with the true Sun's mean motion, the interval between successive passages of it through the first point of Aries is the mean tropical year, or 365·242218 mean solar days. In one mean solar day, therefore, the mean Sun describes an arc of the equator $= \frac{360^0}{365 \cdot 242218}$, which is $= 59', 8 \cdot 33''$. Thus, in a mean solar day a meridian of the Earth moves through an angle $= 360^0, 59', 8 \cdot 33''$: whereas in a sidereal day it moves through 360^0. We can hence compare the lengths of a mean solar and a sidereal day; or, which is the same thing, of a mean solar and a sidereal hour, minute, or second. For a sidereal day is less than a mean solar day in the ratio of 360^0 to $360^0, 59', 8 \cdot 33''$, or about 1 to 1·0027; this gives, for the length of a sidereal day, $23^h, 56^m$ of mean time.

A *sidereal year* is the interval between successive transits of the Sun over a fixed declination-circle, i.e. over one

which meets the ecliptic in a fixed point. The first point of Aries (Chap. VII.) describes in a mean tropical year an arc on the ecliptic of $50{\cdot}23''$. Thus in a mean tropical year a declination-circle through the Sun moves through an angle of $360^0 - 50{\cdot}23''$, whereas in a sidereal year it moves through an angle of 360^0. Also the motion of the first point of Aries is *toward* the Sun; hence, a mean tropical year is less than a mean sidereal year in the ratio of $360^0 - 50{\cdot}23''$ to 360^0, or about 1 to 1·000038.

Again, there is a third kind of year, called the *anomalistic year*;—being the interval between successive passages of the Sun through apogee, or perigee. Apogee moves *forward*, i.e. in the direction of the Sun's motion, through $11{\cdot}25''$ per year. Thus the sidereal year is less than an anomalistic year in the ratio of 360^0 to $360^0, 0', 11{\cdot}25''$, or about 1 to 1·0000087.

131. *Apparent loss or gain of a day in going round the Earth.*

By the motion of the mean Sun in its diurnal circle it comes consecutively over each meridian of the Earth. The absolute time of mean noon will therefore differ at different places. Thus, mean noon at a place west of Greenwich is later, and at a place east is earlier than at Greenwich. And this interval between the absolute times of mean noon at any two places is proportional to the angle by which any fixed meridian of the Earth has separated from a meridian through the Sun, i.e. to the difference of longitude of the places: the interval amounting to 24 hours for a difference of 360^0 of longitude.

Suppose, now, a man to start from the meridian of Greenwich westward: his mean noon will be every day later than at Greenwich by an amount proportional to the longitude of the place he has arrived at. When he has got half-way round the Earth his noon will occur twelve hours after the Greenwich noon. Hence, if he carries a chronometer, keeping Greenwich mean solar time, his chronometer will indicate mean noon, before the local mean noon of the place he has arrived at; and when he is half-way round the Earth, the mean noon by his chronometer will precede the local mean noon by 12 hours. If he con-

tinues his journey in the same direction till he has made a complete circuit and arrives at Greenwich again, his chronometer will have gained 12 hours more; i.e. it will have gained 24 hours or one mean Solar day: in fact the man will have had a number of complete days reckoned by intervals of transit of the mean Sun over his meridian, less by one than the number reckoned by intervals of transit over the meridian of Greenwich. He will therefore seem to have lost a day. Similarly, if he travelled round the Earth from west eastward, he would appear to have gained a day.

132. *Equinoctial time.*

Mean solar and sidereal noon both have reference to the place of observation; and the *absolute* time of noon is different at different places. It is convenient sometimes to have an epoch of time which shall be quite independent of the place of observation: such an epoch is furnished by the time of the Sun's arrival at the vernal equinox in a given year.

The interval of time reckoned in mean solar days, hours, and minutes between this epoch and any succeeding interval of time is called the *equinoctial time* at that instant.

133. *Determination of clock-error.*

In a fixed observatory where there is a transit instrument whose line of collimation moves in the plane of the meridian, and a sidereal clock, we have the sidereal time if we know the error of the clock. For the purpose of determining the clock-error, we may observe the time of transit by the clock either of the Sun, or of one of the 100 stars whose right ascensions are given in the Nautical Almanac for every day of the year. For the R.A. of one of these stars converted into degrees and fractions of a degree will give, by dividing by 15, the number of sidereal hours and parts of a sidereal hour which, at the instant of the star's transit, have elapsed since the transit of the first point of Aries.

The difference between the time so given and the time shewn by the clock is the clock-error. If the Sun be observed, we may in a similar manner, after applying the equation of time, obtain the error of a mean solar clock.

134. *To convert sidereal into mean time and vice versâ.*

We have shewn above how to convert any given *interval* of sidereal time into the corresponding interval of mean solar time, and *vice versâ* (Art. 130). If, however, we wish to find from the sidereal time of any phenomenon the corresponding mean time or *vice versâ*, we must proceed as follows:

(1) To convert *a given sidereal time* into the corresponding mean time.

For this purpose the Nautical Almanac gives every day of the year the mean time of sidereal noon, i.e. of the transit of the first point of Aries.

Now the mean time required is the sum of the mean time of the preceding sidereal noon and of the interval in mean time elapsed since sidereal noon or corresponding to the given sidereal time. We have therefore to extract from the Nautical Almanac the mean time of the preceding sidereal noon, and add the mean interval corresponding to the given sidereal time.

The sum is the mean time required.

(2) To convert *a given mean time* into sidereal time.

The Nautical Almanac gives for every day of the year the sidereal time of mean noon. Adding this to the sidereal interval corresponding to the given mean time we get the sidereal time required.

CHAPTER VI.

ON THE CORRECTIONS TO BE APPLIED TO THE OBSERVED PLACES OF THE HEAVENLY BODIES, ON ACCOUNT OF THE POSITION OF THE OBSERVER ON THE SURFACE OF THE EARTH, AND OF THE PROPERTIES OF LIGHT.

135. *Corrections to the observed places.*

In order to compare with each other observations made at different times and places, it is necessary to know both the motions of the points and planes to which the places of the heavenly bodies are referred, so that the places which they occupied at a given epoch may be known; and also the variations in the apparent directions of the bodies due to variations in the observer's position. When these are known, we are enabled to compare the places of the stars and other bodies with each other as seen at a given time and from a given position.

The apparent place of a body varies at different times and places on account of the position of the observer on the Earth's surface, of the properties of light, and of the change in direction of the Earth's axis.

On account of all these causes *corrections* have to be applied to the observed positions of the heavenly bodies.

In the present chapter we shall discuss the corrections on account of the properties of light and of the observer's position on the Earth; and in the next chapter, those due to the motion of the Earth's axis.

The direction in which a star is seen is determined by drawing a straight line through the eye in such a direction that at the instant when the light from the star arrives at

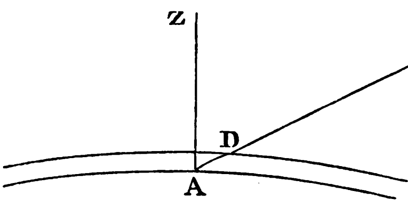
the eye the light is moving along that straight line. If during the whole of its course from the star the light has been advancing to the eye along the straight line drawn from the eye to the star, the star will be seen in the direction of this straight line. This however is not the case for two reasons: the first is, that the Earth and the observer with it are moving in space; light also moves with a uniform and finite though immense velocity; so that if light proceed from a star in the direction of a line drawn from the star to the observer's eye it will be left behind by the Earth and not reach the observer's eye at all. The second reason is that the Earth is surrounded by an atmosphere extending some miles above the surface; this atmosphere is of varying density, being most dense at the surface and rarer the higher we ascend in it; and thus light from a star will not in general describe a straight line, its path being bent or refracted in its passage through the atmosphere.

The correction to be applied to a star's place on account of the atmosphere is called Refraction; and the other correction, due to the motion of the Earth and of light, is called Aberration.

REFRACTION.

136. *Effect of refraction on the apparent position of a star.*

If the Earth were at rest and were of the same temperature throughout its surface, the atmosphere would at every point of the surface have the same pressure and

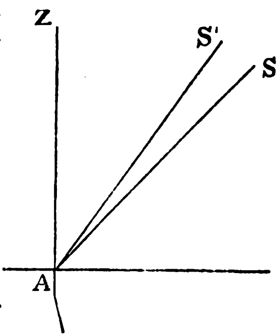


temperature; and at equal distances above the surface the pressure and temperature would be the same, and thus the atmosphere would arrange itself in spherical surfaces all *concentric with* the Earth, and each having the same

pressure and temperature throughout. On account of the unequal distribution of the Sun's heat through the atmosphere and of the rotation of the Earth, the atmospheric pressure and the temperature are different at different points on the Earth's surface; allowance may however be made for this cause of difference, and the refraction then calculated as if each of the concentric layers were of uniform temperature and pressure. We shall, therefore, for our present purpose assume that at all equal heights above the Earth's surface the temperature and pressure are the same. Now the height of the atmosphere is not more than a small proportion of the radius of the Earth; if therefore a ray SD from a star S reaches the atmosphere at D, and the eye of an observer at A, it is clear that, except for great zenith-distances, the distance AD must bear a very small ratio to the radius of the Earth; and the curvature of the strata through which the ray passes will be almost insignificant. If then we suppose the ray to be passing through layers of air contained between planes parallel to the horizon at A, we shall get a result which, for zenith-distances not very great, will be tolerably accurate;—these layers being supposed indefinitely thin, and each of them of the same density throughout.

Now the whole deviation of a ray passing through parallel layers is, by a known optical principle, the same as if the ray had passed directly from vacuum into the last of them.

Let, then, μ be the index of refraction from vacuum into the air at the Earth's surface at A; S the star, AS' the direction of the ray at A; then, $\angle SAS'$ is the refraction; let this $= r$: and let $\angle S'AZ$, the apparent zenith-distance, $= z$: if the ray had been refracted directly from vacuum into the air at A, we should have had for the refraction



$$\sin(z + r) = \mu \sin z \;\ldots (1);$$

and this, by what has been *said, will be* the formula *required in the* actual case.

Since r is small, we have from (1) approximately,

$$\sin z + r\cos z = \mu \sin z;$$

$$\therefore\ r = (\mu - 1)\tan z,$$

r being the refraction in circular measure; if r'' be the number of seconds in the refraction,

$$r'' = \frac{r}{\sin 1''} = \frac{\mu - 1}{\sin 1''}\tan z,$$

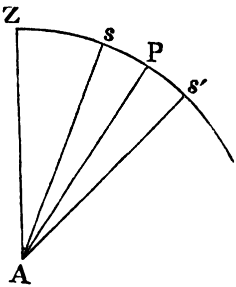
or $r'' = a\tan z$, where a is a constant.

The effect of refraction is, therefore, to raise the position of a star nearer to the zenith, in the vertical plane through the star, by an angle which varies as the tangent of the apparent zenith-distance.

137. *To determine the coefficient of refraction.*

We shall now shew how the constant a, which is called the *coefficient of refraction*, may be determined.

This is done by observation of circumpolar stars, i.e. of stars very near the pole.



Let P be the pole, Z the zenith, s and s' the points in which the diurnal circle of a circumpolar star meets the meridian; then $sP = s'P$.

An observer at A will however see the star in a position nearer Z by the angle through which it is raised by refraction.

Hence, if z be the *observed* zenith-distance of s,

$$Zs = z + a\tan z;$$

so, if z' be the observed zenith-distance of s',

$$Zs' = z' + a\tan z'.$$

But $\quad Zs = ZP - Ps,$

and $Zs' = ZP + Ps$; for $Ps' = Ps$;

$$\therefore\ z + a\tan z + z' + a\tan z' = Zs + Zs' = 2ZP.$$

So, if another star be observed and have for its observed zenith-distances z_1 and z_1',

$$z_1 + a\tan z_1 + z_1' + a\tan z_1' = 2ZP.$$

Equating these two values of $2ZP$, we have

$$z + z' + a(\tan z + \tan z') = z_1 + z_1' + a(\tan z_1 + \tan z_1');$$

8—2

from which equation a may be calculated. By numerous observations a has been determined to be $= 57{\cdot}5''$.

The refraction, it must be remembered, is in most cases very small, being, at an altitude of 45°, rather less than 1′.

The above formula for refraction, though a fair approximation, is by no means sufficiently accurate for Astronomical purposes; but it may serve to illustrate the way in which allowance is made for it in obtaining the true place of a star from the apparent, and *vice versâ*.

138. *Tables of refraction.*

Instead of finding the value of tan z for each observation, and multiplying it by its coefficient a, the values of tan z, or rather of the function which stands in its place in the rigorous investigation, are found for every *degree* of angle from 0° to 90°. Each of the values so found, being multiplied by the Constant of Refraction, gives the corresponding Refraction. We are thus enabled to form a *Table of Refractions*. In practice it is necessary, since the refraction becomes more irregular the greater z is, to tabulate the refraction for intervals of less than a degree for zenith-distances greater than a certain angle.

The Tables of Refraction now generally used are those of the illustrious Bessel. They give the refraction for temperature 50° F. and pressure 29·6 inches for *every degree* of zenith-distance from 0° to 35°; from 35° to 54° *for every* 30′; and for still smaller intervals for angles below 54°; for altitudes less than 10° the refraction is given *for every* 5′. There are also tables giving the corrections to be applied to each refraction for a temperature and pressure different from those taken as the standard.

139. *Various effects of refraction. Twilight.*

Since by refraction the zenith-distances of all the heavenly bodies are diminished, they all are seen on the horizon before the actual time of rising, and after the actual time of setting. The whole period during which they appear above the horizon is thus increased. For example, the Sun appears above the horizon a short time after it has actually set.

When the Sun is near the horizon, it appears of an oval shape, its circumference being somewhat in the form of an ellipse whose least axis is vertical. This arises from the

fact that, the refraction being greater the greater the zenith-distance, the lower limb of the Sun is more raised than the upper; the effect being a contraction of all vertical lines on its apparent surface, while the horizontal lines are nearly unaltered.

Another effect due to the Earth's atmosphere is that, even when the Sun is below the horizon, rays from the Sun will pass through and illuminate that part of the atmosphere which is above the observer's horizon. By this direct illumination of the atmosphere by the Sun, and by the reflexion from particles floating in the atmosphere, the Sun's light still remains for some time after sunset. This effect is called twilight. It is found that twilight lasts at any place during the time which elapses between sunset and the arrival of the Sun at a position on its diurnal circle 18° of zenith-distance below the horizon of the place.

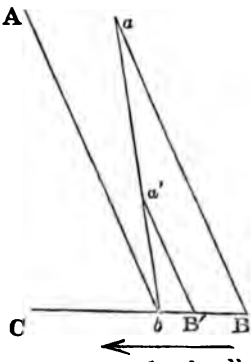
ABERRATION.

140. *Amount and coefficient of aberration for stars.*

We now proceed to the correction called *Aberration:* this correction is due to the motion of the Earth and of light; in virtue of which two motions the line joining the observer's eye and the star is not the direction in which light comes to him from the star.

To estimate the amount of aberration for any star.

Let BC be the tangent to the Earth's orbit at b: Bb an arc of the orbit described in a small interval of time, say 1 second; then Bb may be considered to coincide with the tangent at b, and to be described uniformly with the velocity the Earth has at b. Also let ab be the direction of the ray from a star which arrives at the observer's eye at b; the light having arrived at a when the Earth was at B. Then, we can shew that a line through the observer's eye parallel to aB will, as the observer is carried by the Earth from B to b, be the direc-



tion in which light comes from a to his eye while he is moving from B through b.

For, let $B'a'$ be any intermediate position of Ba, cutting ba in a': then,

$$\frac{BB'}{aa'} = \frac{bB}{ba} = \frac{\text{velocity of the Earth}}{\text{velocity of light}},$$

and therefore aa' is described by the light in the same time as BB' by the Earth; and thus the light continually arrives simultaneously at successive points of the fixed line ab, and of the line aB—which is fixed with respect to the observer, but is carried on with him by the Earth's motion.

Hence the line Ab is the straight line drawn from the observer's eye along which the light from the star moves at the instant of arriving at his eye. It is therefore the direction in which the star is seen.

The actual direction of the star is ba; that is, the direction of the star *was ba at the instant the light left it which reaches the observer at b*: and the apparent direction is bA, drawn parallel to Ba. The aberration is therefore the angle Aba, which is equal to the angle baB: and

$$\frac{\sin baB}{\sin CBa} = \frac{bB}{ba} = \frac{\text{velocity of the Earth}}{\text{velocity of light}}:$$

whence, since the aberration is always very small,

$$\text{aberration} = \frac{\text{vel. of Earth}}{\text{vel. of light}} \cdot \sin CBa$$

in circular measure; hence the number of seconds in it

$$= \frac{\text{vel. of Earth}}{\text{vel. of light}} \cdot \frac{\sin CBa}{\sin 1''}.$$

The angle CBa made by the direction of the Earth's motion with the direction of the star is called the *Earth's way:* hence the aberration varies as the sine of the Earth's way, and is greatest when this angle is 90°. Thus at the pole of the ecliptic the aberration has its greatest value in all positions of the Earth in the ecliptic.

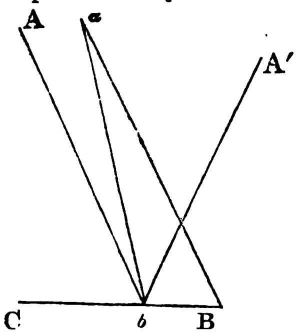
The constant multiplier is called the *coefficient* of Aberration.

141. Aberration for Sun, Moon, and planets.

In the case of the Sun, Moon, or a planet, we can

obtain the whole correction for aberration; i.e. from the apparent place we can find the actual place *at the instant of observation.* For, in the figure we may suppose a to be the actual position of the body—a planet, suppose—at the moment when the ray starts from it which reaches the observer at b, and Bb the small arc described by the Earth in the time the ray takes in describing ab.

Now let A' be the actual place of the planet at the instant of observation. Then bA' is the true direction of the planet when the Earth is at b; and bA, parallel to Ba, is, by what precedes, the direction in which it is seen. The whole aberration is therefore equal to the angle AbA'; but this angle is equal to the angle between bA' and aB, which is the angle between the positions of the line joining the Earth and the planet at the instant of observation and at the instant when the planet was at a: and in this interval light has travelled from the planet to the Earth.



Now light travels over a space equal to a radius of the Earth's orbit in $8^m. 18^s$. If the Sun, therefore, be the body, the Sun's apparent direction is the direction it really had $8^m. 18^s$ previously; if the Moon, or a planet whose distance from the Earth, in terms of the Sun's distance as unity, is D, it is the direction it had $8^m. 18^s \times D$ previously. If therefore t' be the time of observation, and $t' = t + 8^m. 18^s \times D$, the observation will give the true place for the time t. In order to correct, therefore, the place of the Sun, Moon, or any planet for aberration, we have simply to correct the time of the observation by the interval $8^m. 18^s \times D$.

Since the motion of the Earth as well as of the planet must be taken into account, the position of the planet must be referred to the Earth as origin. The result will be the *true geocentric* place at the time so corrected.

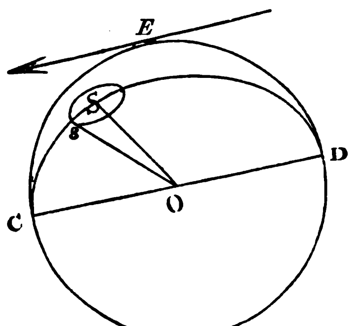
142. *Effect of aberration on the apparent position of a star in the course of a year.*

We will now explain the effects of aberration on the apparent place of a star in the course of a year.

The place of a *star*, when corrected for aberration, is (Art. 140) the place it occupied some time before the observation; the interval of time being such that during that interval light travels the distance of the star from the Earth. We will suppose that the true or corrected place will not change in the course of the year: that the star in fact is so distant that lines joining its position at any time with all points in the Earth's orbit may be considered parallel; this is the same as saying that the star has no sensible annual parallax (Art. 160). If the star has a parallax, the effect of it on the place of the star can be separately allowed for.

From Art. 140 it is seen that the star is depressed by aberration towards the direction of the Earth's motion, in the plane through that direction and the star; assuming the Earth's orbit to subtend no appreciable angle at the star, we may take this plane to coincide with a plane through the star and a line through the Sun parallel to the Earth's direction of motion.

Let, now, E be the position of the Earth in its orbit at any time, S the true place of a star, CD a diameter of the



Earth's orbit (which we shall suppose circular) parallel to the tangent at E: then S is displaced to s, on the great

circle of the celestial sphere formed by its intersection with a plane through CD and S; and s is between S and C. As E moves round the orbit, CD revolves in the plane of the orbit through 360°, and s moves round S till CD comes back to its original position: s will then be in its old position, having described a closed curve about S.

Since (Art. 140) aberration varies as the sine of the Earth's way, the angle SOs varies as sin SOC,—the direction of the star from O being supposed identical with that from E, or the star being considered infinitely distant;—thus the aberration will vary with the position of the star, and also with the position of the Earth in its orbit, being greatest when the direction of the Earth's motion is perpendicular to the direction of the star.

143. *Discovery of aberration by Bradley.*

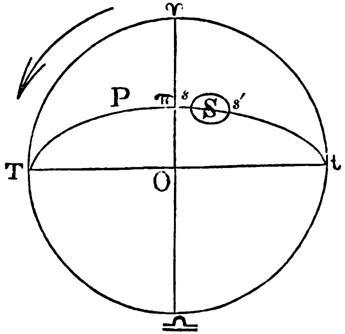
We will now briefly describe the process by which Bradley in the year 1726 detected and explained this apparent motion of the stars.

He was making observations, in the plane of the meridian, of a star called γ Draconis, for the purpose of detecting, if possible, annual parallax. This star was chosen because it was a bright star and passed the meridian within an angle of a few minutes to the south of his zenith; refraction was consequently extremely minute, and therefore any error to which it was liable must have been too small to affect the result of his observation. The star was also very nearly in the solsticial colure, and distant from the pole of the ecliptic—where the aberration is greatest—only 15°.

In the figure (next page), let π be the pole of the ecliptic, P of the equator, S the star, Tt the intersection of the solsticial colure with the ecliptic. Then, when the Earth is at T, the Sun is seen in direction Tt at its greatest distance from the pole P; and when the Earth is at t, it is seen nearest P. Hence T and t are respectively the positions of the Earth at the winter and summer solstices. So ♈ and ♎ (90° from T and t) are the positions of the Earth when the Sun is at the autumnal and vernal equinoxes.

Let the small curve which the apparent place of the star describes about S meet the solsticial colure in s and s';

then s, s' are the apparent places of the star when the Earth is at ♈, ♎; and as the Earth moves through ♈ in



direction of the arrow-head, s moves round S along the circumference of its aberration-curve. When the Earth is at ♈ it is moving in a direction parallel to tT, and the star is then seen at s; and as the Earth moves in its orbit s moves along its curve in a direction perpendicular to the plane TPt. At T and t the apparent places of the star are the points of intersection of the plane ♈S♎ with the aberration-curve; and the apparent motion of the star due to aberration is then parallel to the plane TPt; thus when the Earth is at either solstice the apparent change of N.P.D. of the star on account of aberration is most rapid. Similarly it may be shewn that when the Earth is at either equinox the change of N.P.D. of the star is least.

Bradley observed the star first of all in December, 1725, when the Sun was near the winter solstice; the Earth was therefore near T, and the apparent motion of the star in N.P.D. was most rapid. His observations were made with a zenith-sector, which gave him the star's meridian zenith-distance for every transit of the star. Since the star's apparent N.P.D. was daily increasing, the zenith-distance was increasing, and the star at each successive transit was found to move more and more towards the south: this went on till about the beginning of March, 1726, when the Earth being near ♎ the star appeared near s'. After

that its N.P.D. altered very slowly and it appeared nearly stationary. About the middle of March its N.P.D. diminished, and the star appeared to move backwards towards the north until September, when it remained nearly stationary for some days; it then moved back again, till in December it returned to the position in which it was when Bradley first observed it. It was shewn by Bradley that all these motions could be accounted for by aberration with tolerable accuracy, assuming as a coefficient of aberration 20·25″. He was induced to disregard the hypothesis of annual parallax by observing that the change of N.P.D. of the star was greatest when it would have been least if parallax had been the cause, and *vice versâ*.

For further information the student is referred to Grant's *History of Physical Astronomy*, page 336.

144. *Aberration-curve an ellipse.*

The aberration-curve can easily be shewn to be approximately an ellipse with S as centre, and ss' as minor axis. For since (Art. 140) $\frac{bB}{ba}$ is a nearly constant ratio, the velocity of the Earth in its orbit being nearly constant, any section, parallel to the ecliptic, of the cone described by bA about ba is a circle: and we may consider the section of this cone by the celestial sphere as approximately a plane section, and therefore, in general, an ellipse. Also, the angle SOt is the least of the angles which SO makes with lines in the plane of the ecliptic: hence the aberrations in the plane TSt are the least, and ss' is the minor axis of the aberration-ellipse.

145. *Velocity of light ascertained from eclipses of Jupiter's satellites.*

Before the discovery of aberration, it had been observed that the eclipses of Jupiter's satellites occurred sometimes earlier and sometimes later than the time calculated. Römer, to account for this phenomenon, had assumed that this was due to the less or greater time taken by the light to come from Jupiter to the Earth, according to its distance; the distance being least when Jupiter was in opposition, and greatest when in conjunction with the Sun. The uniform velocity of light which Römer was

obliged to assume to reconcile the observed times of the eclipses with the calculated times, when substituted in the expression for aberration, gives the same value for the coefficient as that obtained by Bradley by the observations of γ Draconis, thus confirming, from quite independent considerations, the theory of aberration by which Bradley explained the apparent motion of that star.

146. *Diurnal aberration.*

In discussing the aberrations of stars we have hitherto taken account only of the orbital motion of the Earth: we must however combine with the aberration so obtained a small correction due to the Earth's diurnal motion. This will be different for different latitudes on the Earth's surface. It is called the *diurnal aberration;* its effect on a star, at the time of transit, is to displace it in a direction perpendicular to the meridian.

To calculate the coefficient of diurnal aberration for a given latitude λ.

Let r be the Earth's radius; a that of the Earth's orbit: then, since a place in latitude λ describes a circle of radius $= r \cos \lambda$, in one revolution of the Earth, we have, taking a second as the unit of time,

$$\text{velocity of the place} = \frac{2\pi r \cos \lambda}{24 \times 60 \times 60}$$

$$\text{velocity of light} = \frac{a}{8 \times 60 + 18},$$

$$\frac{1}{\sin 1''} = 206265;$$

$$\text{also } \frac{r}{a} = \frac{1}{23750} \text{ nearly.}$$

$\therefore$ coefficient of diurnal aberration

$$= \frac{206265}{23750} \cdot \frac{2\pi \times 498}{86400} \cos \lambda$$

$$= 0{\cdot}3'' \cos \lambda \text{ nearly.}$$

PARALLAX.

147. *General effect of parallax.*

The direction in which a star is seen by an observer on the Earth's surface would, if the stars were not at an immeasurable distance, be appreciably different when the

star is seen from different points on the Earth's surface; and it would therefore be necessary to refer them to some common point from which their directions should be estimated.

For the fixed stars, however, the angles subtended even by a diameter of the Earth's orbit are, for the most part, absolutely inappreciable by the most refined observations. This is not the case with the Sun, Moon, and planets, lines from which to different points of the Earth's surface differ perceptibly in direction. It is therefore necessary to refer the observations of these bodies to some one common point. The centre of the Earth is the point to which they are all referred.

Let C (fig. next page) be the centre of the Earth, O the position of an observer on it, M of the centre of the Sun, Moon, or a planet; then the $\angle OMC$ is called the *parallax* of M.

If the Earth were a perfect sphere, the $\angle MOZ'$ made by MO with CO produced would be the true zenith-distance of M. Since, however, the Earth is a spheroid, generated by the revolution of an ellipse of small ellipticity about its minor axis, $Z'OC$ is not the normal at O: let ZO be the normal; then ZO lies in the meridian through $Z'O$, and makes a small angle with it. Also, since the ellipticity of the Earth is correctly known, the angle between ZO and $Z'O$ can be calculated for every place whose latitude is known. If then the zenith-distance of M be observed at O, we have, by subtracting the angle ZOZ', the angle $Z'OM$.

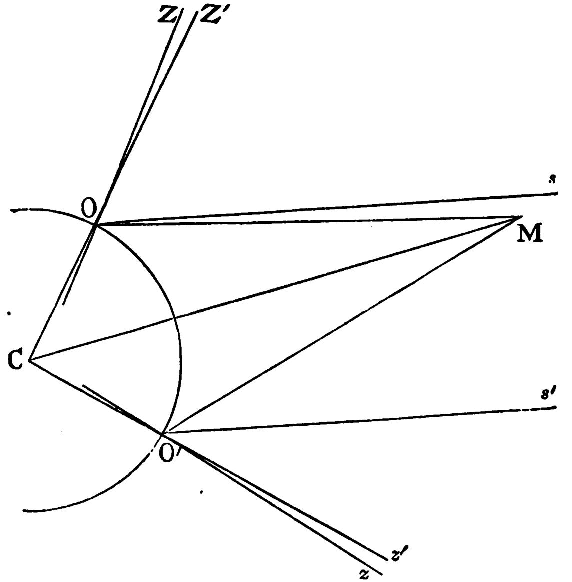
The general effect of parallax is, as is obvious from the figure, to increase the zenith-distance, and therefore to depress a body in a vertical plane through the star. It is zero at Z', or very near the zenith, and greatest when $\angle COM = 90°$, or when the body is on the horizon.

148. *Parallax of the Moon determined by observations at two places on the same meridian.*

We will now shew how by observations of the Moon at different places on the same meridian, whose latitudes are known, we can find its distance, and its parallax at any zenith-distance.

Let O, O' be the two stations from which the observations are taken. Produce CO' to z', and let zO' be the

normal to the Earth at O', then zO, zO' are both in the plane COO', which is the meridian of the two places.



On the same day, let observations be taken with the Mural Circle (or Transit Circle) at the transit of the Moon over the meridian. We thus obtain the angles ZOM, $zO'M$, whence the angles $Z'OM$, $z'O'M$, and therefore their supplements COM, $CO'M$, are known. Let $COM = z$, and $CO'M = z'$, and let r, r' be the radii of the Earth at O and O', which are known, since the latitudes of O and O' are known. And let the geocentric distance of the Moon, $CM, = R$.

Then, if p, p' be the Moon's parallaxes at O and O', the triangle COM gives

$$\frac{\sin p}{\sin z} = \frac{r}{R} \quad \text{.......................} \quad (1),$$

similarly,

$$\frac{\sin p'}{\sin z'} = \frac{r'}{R} \quad \text{.......................} \quad (2).$$

Again, the angle OCO' can be determined in terms of the latitudes at O and O': let it equal a; then

$$p + p' + z + z' + a = \text{angles } OMO', OCO', COM, CO'M,$$
$$= 360^0 \text{ .... (3).}$$

Equations (1), (2), and (3), determine R, p, and p'.

Having determined R, equation (1) gives p the parallax for any point on the Earth's surface for which r is known; i. e. for any place whose latitude is known.

149. *Method of obviating the uncertainty of refraction.*

We have supposed that the zenith-distances at O and O' can be correctly observed; unfortunately, the uncertainty of refraction would seriously affect observations in which such extreme accuracy is required as in the present case. In equations (1) and (2), any error in z, z' resulting from refraction will be unimportant, since it will be multiplied by the small fractions $\frac{r}{R}$, and $\frac{r'}{R}$: in equation (3) however, it would materially affect the determination of $p+p'$. It is important, therefore, if possible, to determine $p+p'$ by some method in which the error of refraction may be as nearly as possible got rid of. This is done by observing some star which passes the meridian at the same time as the Moon, and very near it.

Draw Os, $O's'$ in the direction of such a star; these lines are sensibly parallel. The *difference* of the observed zenith-distances of s and M at O and O', gives the angles sOM, $s'O'M$. And since the star and Moon differ little in zenith-distance, any error in the refraction will affect both nearly equally; the angles sOM, $s'O'M$ are therefore nearly unaffected by any error due to this cause.

Since sO, $s'O'$ are parallel, the sum of the angles sOM, $s'O'M$ is the angle OMO' or $p+p'$. Thus $p+p'$ is determined very accurately: let it equal β; we may therefore write

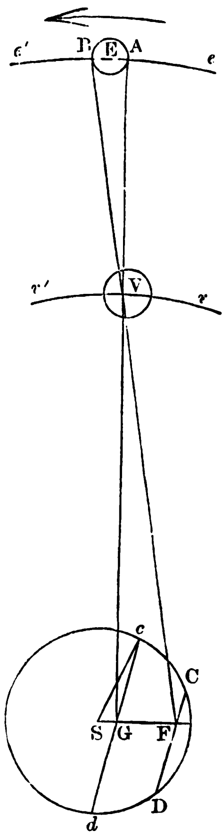
$$p+p' = \beta \text{ .... (4),}$$

and equation (4) may be substituted for (3).

It is clearly desirable that O and O' should be very distant from each other. The observatories at Greenwich and at the Cape of Good Hope satisfy this condition, the north latitude of Greenwich being $51^0.28$, and the south

latitude of the Cape being 33°. 56′. The parallax of the Moon has been determined by observations taken at these observatories: R was found to be about 240,000 miles or about 60 radii of the Earth; and the value of p when $z = 90^0$—which is called the *horizontal parallax*—about 57′.

150. *Sun's parallax by transit of Venus.*



This method cannot be used for the Sun; a more refined method being necessary on account of its great distance and consequently small parallax.

One of the best methods of determining the Sun's distance and parallax, is by observations at two observatories, a great distance apart, of transits of Venus across the Sun's disc.

If the orbit of Venus were in the plane of the ecliptic, Venus would transit the Sun's disc at each conjunction. Since, however, its orbit is inclined to the ecliptic, a transit can only occur when at the time of conjunction Venus is sufficiently near the ascending or descending node of its orbit.

The actual determination of the Sun's parallax by this means is so extremely complicated, that it would be nearly impossible to give a satisfactory account of it in an elementary manner, if all the minute details which are incidental to it were taken into account. The general principle of the method may however be explained without much difficulty.

Let E, V, S be the positions of the centres of the Earth, Venus, and the Sun at a time when Venus is in conjunction, and is near its node. The orbit of Venus being inclined at a small angle, about 3°. 23′, to the ecliptic, if Venus be sufficiently near its

node its centre will be distant from the Sun's centre by less than the angular radius of the Sun: it will therefore be seen projected as a dark spot on the Sun's disc.

At this instant, suppose A and B to be the positions of observers, A in the northern hemisphere, and B in the southern, at two places at the extremities of a diameter of the Earth. Suppose, also, that the diameter is the one which is at that time perpendicular to the orbit of Venus. Since the inclination of the orbit of Venus is small, AB will be nearly perpendicular to the ecliptic.

At some time—about three hours—before Venus is actually in conjunction, it will appear at the edge of the Sun's disc on the east side of the centre; and will move across the disc from east to west. For, in passing through conjunction, the Earth and Venus will each describe a portion of its orbit, the velocity of Venus being greater than that of the Earth: the motion of Venus relatively to the Earth will therefore be in the direction of Venus's orbital motion, which is from east to west at inferior conjunction.

Its apparent path across the Sun, as seen from the Earth's centre, will be parallel to its direction of motion; as seen from A and B, it will therefore appear to describe two lines CD, cd, parallel to the direction of motion of Venus, and therefore in the case supposed, nearly perpendicular to AB. Let the plane AVB meet CD, cd in F and G; then FG is parallel to AB.

Now $$FG : AB = VF : VB;$$
and the ratio of the distances of Venus and of the Earth from the Sun is known by Kepler's third law; thus, the ratio of VF to BF is known, and hence the ratio of $VF : VB$. The latter ratio is about 5 : 2.

Hence FG is $2\frac{1}{2}$ times AB; and therefore the angle subtended by FG at E is about $2\frac{1}{2}$ times the angle subtended by AB at S, i.e. it is 5 times the Sun's horizontal parallax.

If, then, by any means we can measure FG, we shall obtain 5 times the Sun's horizontal parallax. Only about one-fifth, therefore, of any error in estimating FG will affect the determination of the Sun's parallax. We have, in fact, 5 times as great an angle to measure.

Now the angle subtended by FG at the Earth's centre

cannot of course be found by direct measurement. It is however inferred from the times occupied by Venus in performing its transit as observed from A and B: and those times of transit being about 6 hours each, the accuracy with which they can be determined is very great, an error of a second of time being a very small proportion of the whole.

From the tables of the motions of Venus and the Earth, we can calculate the time in which the geocentric motion of Venus would be an angle equal to a diameter of the Sun. Comparing this time with the times of describing CD and cd, we have the ratios of cd and CD to the Sun's diameter. These two ratios determine the positions of cd and CD on the Sun's disc; and hence FG is known.

The method of observing the time of ingress of Venus's centre, is by taking the mean of the times when Venus just touches the disc and when it is just wholly within it; and the time of egress is found in a similar manner. The difference of the times so found is the duration of the transit.

In the rigorous investigation many circumstances which we have entirely omitted, such as the inclination of AB to a perpendicular to the ecliptic, the exact motions of Venus and the Earth, and the rotation of the Earth during the transit, must be taken into account. The above can therefore only be looked upon as a sketch of the principle of the method, as applied to a much simpler case than occurs in practice. The rotation of the Earth is small compared to its orbital motion, but it perceptibly affects the times of ingress and egress of Venus.

It is easily seen that the rotation increases the duration of the transit to an observer at A, and diminishes it to an observer at B; for the rotation moves A and B in opposite directions.

The Sun's parallax as determined by transits of Venus has been estimated at 8·57″. This value makes the Sun's distance equal to about 95 millions of miles.

151. *Reasons for preferring transits of Venus to transits of Mercury.*

The same method is applicable to transits of Mercury; *but transits* of Venus give much more trustworthy results, *for many reasons.* In the first place, Mercury is much

farther from the Earth at inferior conjunction than Venus is; the parallax of Mercury, on the effect of which the difference in the observed times of transit principally depends, is therefore much less; the difference in the observed times of transit at the two stations is therefore much less for Mercury than for Venus, and therefore any error in estimating the times more seriously affects the result.

Again, the difference of the volocities of Mercury and the Earth is much greater than that of Venus and the Earth; the apparent motion of Mercury across the Sun's disc is consequently much more rapid; the time of transit being shorter, any error in determining it is therefore a larger proportion of the whole, and is thus more important.

Another objection to Mercury is that its apparent diameter is much less, so that the moments of internal contact cannot be observed so accurately.

152. *Intervals at which transits of Venus may occur.*

In order that a transit of Venus may happen near either node it is necessary not only that Venus should be near that node, but that the Earth should be at the time in nearly the same direction from the Sun as Venus is. Thus in the interval between two transits of Venus, not only Venus but also the Earth must have completed almost exactly an integral number of sidereal revolutions. Let then m and n be the number of revolutions completed respectively by Venus and the Earth between two transits; then, since a sidereal revolution of Venus is effected in 224·700, and of the Earth in 365·256 mean solar days, we must satisfy as nearly as possible the equation

$$m \times 224{\cdot}7 = n \times 365{\cdot}256,$$

$$\text{or } \frac{224{\cdot}7}{365{\cdot}256} = \frac{n}{m};$$

and the integers which most nearly satisfy this equation are found by finding the converging fractions to the fraction on the left-hand side. These are found to be

$$\frac{8}{13}, \frac{237}{382}, \frac{713}{1159}:$$

thus the number of years elapsed between transits at the *same node* are 8, 237, 713, or multiples of these numbers.

At the present time the Earth passes through the line of nodes of Venus's orbit in June and December. Transits of Venus can therefore happen only in these months. The next transit will occur at the ascending node in December, 1882.

153. *Sun's distance by parallax of Mars in opposition. Distances of the planets deduced by Kepler's third law.*

Another method of determining the Sun's distance is by finding the parallax of Mars when this planet is in opposition. In this position it is nearest to us, its distance being the difference of the radii of the orbits of Mars and the Earth. Its parallax is then about 22″, which is large enough to be determined by a method similar to that used for the Moon. We thus obtain Mars' distance, or the difference of the radii of the orbits of Mars and the Earth. And by Kepler's third law we can find the ratio of these radii; we thus deduce the radius of the Earth's orbit, i.e. the Sun's distance. The Sun's parallax, as determined by observations of Mars, has been estimated at 8·95″.

M. Foucault has recently by direct experiment determined the velocity of light. Assuming the correctness of his result we can determine, from the formula for the constant of aberration, the velocity of the Earth. Now the length of a sidereal year is very accurately known: hence knowing the velocity of the Earth we can find the length of the circumference of its orbit: and this being equal to $2\pi \times$ Sun's distance, the Sun's distance can be inferred. The distance thus determined is about 92 millions of miles: being 3 million miles nearer than that given by the transit of Venus. It gives for the Sun's parallax 8·86″. This is nearly the parallax which M. Le Verrier had been led to adopt from considerations quite independent.

Having determined the *Earth's* distance from the Sun, the distances of the other planets from the Sun become known by Kepler's third law. Hence we can infer the distance of each planet from the Earth at any part of its orbit, and consequently its parallax.

CHAPTER VII.

PRECESSION AND NUTATION.

154. *Precession and Nutation due to change of direction of the Earth's axis.*

The motions, called *Precession and Nutation*, are real motions of the poles and equator of the Earth. The Earth's axis of rotation moves, in the course of a year, in such a manner as to maintain almost exactly the same direction in space; it moves in fact very nearly parallel to itself. The equator, being perpendicular to it, moves also very nearly parallel to itself. And the line of intersection of the equator and the fixed plane of the ecliptic consequently maintains very nearly a fixed direction. If the axis of the Earth moved *exactly* parallel to itself, the R.A.s and N.P.D.s of the fixed stars would be invariable. This however is not the case. The R.A.s and N.P.D.s are found to be subject to variation. We infer therefore that the Earth's axis is not fixed in direction.

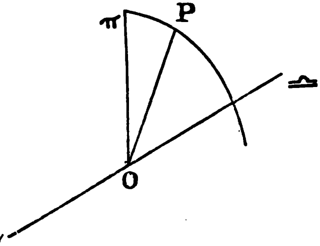
PRECESSION.

155. *Nature of the motion called Precession. How first detected.*

When the R.A.s and N.P.D.s of any star from time to time are converted into celestial longitude and latitude, so as to refer the star's place to the ecliptic, it is found that the longitude of the star increases uniformly with the time, while the latitude is unaltered. This proves that the ecliptic is fixed, but the line of intersection of it with the equator *regredes* uniformly. Since the Sun's motion is

progressive, the intersection of the ecliptic with the equator moves so as to meet the Sun. The Sun therefore crosses the equator *earlier* than it otherwise would. On this account the motion is called *Precession* of the Equinoxes. The obliquity of the ecliptic is found to be subject to no increase or diminution with the time. The position of the equator being given at any time, its position at any subsequent time is determined by drawing a plane at the same inclination to the ecliptic through the new position of the line of intersection.

The inclination of the planes being unaltered the inclination of their poles, which is the same angle, is unaltered.



Let now O be the centre of the celestial sphere; ♈O♎ the line of the equinoxes; π, P the poles of the ecliptic and equator. Then πO and PO are both perpendicular to ♈♎; and the plane πOP is therefore perpendicular to ♈♎. The angle through which ♈♎ moves in any time is therefore the angle through which plane πOP moves about $O\pi$. Hence P describes a small circle uniformly about π. The angle through which ♈♎ regredes along the ecliptic in one year is about 50·2″; the pole of the equator will therefore complete a revolution about the pole of the ecliptic, and the line of the equinoxes will complete a revolution in the ecliptic, in about 25,820 years. The obliquity of the ecliptic is about 23°. 28′; hence, at the beginning and end of an interval equal to half this time, P will be in positions about 47° apart: thus, the pole now occupies a position 47° apart from the position which it occupied about 12,910 years ago.

At the present time the north pole is about $1\frac{1}{2}°$ distant from a star of the second magnitude in the constellation of the Lesser Bear; which star, on account of indicating so nearly the position of the pole, is called the *pole-star*, or *Polaris*. This star will, 12,910 years hence, be about half way between the pole and the equator.

The equinoctial points were once in the constellations of Aries and Libra. In consequence of the regression of the equinoctial points on the ecliptic, the point which was in Aries, and was hence called the first point of Aries, is now entering into the constellation Pisces. It will in the course of time pass through all the constellations in the Zodiac, and in the lapse of ages return to Aries.

The effect of the regression of the equinoctial points in diminishing the length of a year has been explained in the Chapter on Time:—(Chapter v.). Since the seasons depend on the positions of the Sun with respect to the equator, the effect of the motion of the equinoxes to meet the Sun is to shorten the interval at which each recurs.

In 72 years the equinoctial points will have regreded through more than 1°; if, therefore, the R.A.s of stars be compared at intervals of a century or more, the apparent motion in R.A. becomes quite conspicuous. It is in this manner that Precession was first detected (about 150 B.C.), long before the time of refined observations.

156. *Physical cause of Precession.*

The physical cause of Precession was discovered by Newton. It was shewn by him to arise from the attractions of the Sun and Moon on the spheroidal Earth. If the Earth were a sphere the attraction of each would pass through the centre of the Earth and produce no motion of the axis. But since the Earth consists, in fact, of a sphere about the polar axis as diameter, in addition to a portion due to the excess of the equatorial diameter of the Earth's spheroid over the polar diameter, there is an attraction on this protuberant part, which produces a motion of the axis the general nature of which has been described. The part of the Precession due to the Moon is called *Lunar Precession*, and that due to the Sun *Solar Precession*. The Moon, though much smaller than the Sun, yet, in consequence of being so near the Earth, produces by far the most important part of the effect; the combined effect of the two is called *Lunisolar Precession*.

157. *Stars' places corrected for Precession.*

The places of the stars are corrected for Precession by *referring* them to the positions which the equator and

the first point of Aries had at some fixed epoch. The motions of this plane and point among the fixed stars having been determined by observations of the stars throughout a long period, we are able, given the position of a star with reference to the positions of the equator and first point of Aries at the time of observation, to determine the position with reference to the places which they occupied at a fixed epoch—say at the beginning of the year 1800.

NUTATION.

158. *Effect of Nutation on the places of stars. Physical cause of Nutation.*

The motion of the Earth's axis which we have described is, though not quite its accurate motion, so near to it, that the difference could not be detected except by observations capable of considerable accuracy. It was found however by Bradley, that after correcting for Precession there was still an apparent motion of the stars to be accounted for, small indeed, but quite discernible with the instruments then in use.

By observations extending over a period of 19 years, he found that the stars had all apparent very small motions by which they deviated from their original positions, and finally returned to them again. He inferred from this that the position of the pole varied slightly from the place which it would occupy on account of Precession, but that its motion with reference to that place was a motion of oscillation in virtue of which it returns periodically to its mean position, the period of the oscillation being about 18½ years. This period corresponding accurately with the period of a revolution of the Moon's node confirmed Bradley in the original hypothesis by which he accounted for the motion, which was that it was produced by the varying attraction of the Moon on the protuberant matter of the Earth in different positions of *the Moon's orbit* with respect to the equator.

This oscillation of the axis about its mean place is called *Nutation;* and the correction necessary to be applied to the places of the star in consequence of it, in order to refer *them to* planes absolutely fixed, is the *correction for Nutation.*

This account of the phenomenon has been fully confirmed, not only by observation, but by the agreement of its period and amount with that given by an investigation of the action of the Moon on the Earth on dynamical principles.

Strictly speaking the whole Nutation is due to the combined actions of the Sun and Moon. But the action of the Sun in producing Nutation, though perceptible, is very much smaller than that of the Moon: its period is half a year.

The maximum change in the obliquity of the ecliptic produced by Nutation is about 9″: and the maximum change in the longitudes of stars in consequence of the nutational motion of the first point of Aries is nearly 17″.

The actual motion of the pole of the equator about the pole of the ecliptic will, on account of Precession and Nutation, be in an undulating curve separating very little from a small circle about the pole of the ecliptic, and such that, if the position of the pole of the equator be given at any time, its positions at intervals of about $18\frac{1}{2}$ years from that time will all lie in the same small circle.

159. *Motion of the ecliptic. Proper motions of stars.* The ecliptic has been assumed hitherto to be an absolutely fixed plane. It however has a small motion due to the action of the planets on the Sun. This motion is in reality periodic: but its period is so immense that it may practically be assumed to be non-periodic. The effect of it is a gradual small change in the obliquity and in the position of the first point of Aries. These effects are called Planetary Precession. Being produced by a motion of the ecliptic, they do not affect declinations of stars: but the R.A.s are continually increased by it to the amount of less than 2″ per year.

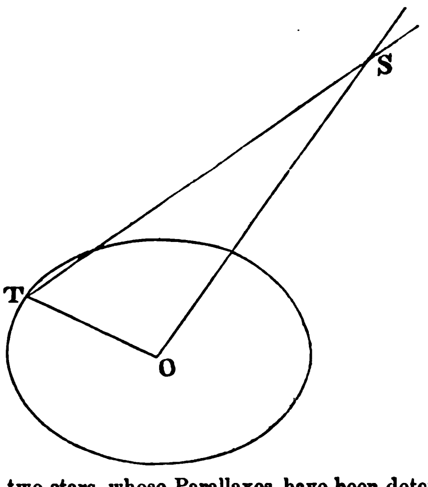
Even after allowance has been made for all these causes of apparent displacement of the stars, they are still found to be generally subject to minute apparent motions among one another: these are called *Proper motions* of the stars, and are supposed to arise either from actual motions of their own, or apparent motions due to an actual motion of the solar system in space. Such motions however are, as may be supposed, either

continuous in one direction, or of such long periods that we may practically consider them to be so.

160. *Stellar—or annual—parallax.*

We now come to the question of stellar Parallax. The Parallax of a star can only be detected by its apparent displacement as seen from different points of the Earth's orbit. The greatest effect thus produced is when the star is observed from opposite points of the orbit; it is in all cases extremely small, and only in a very few has it been detected with certainty at all. It will readily be understood that, after the host of corrections which have to be applied to a star's place in order to refer it to planes absolutely fixed, the minute amount of Parallax will stand a great chance of being absorbed in some small error in the application of one or other of these corrections.

Two considerations serve as guides in selecting stars to observe for Parallax. One is the brightness, which is an *à priori* argument of proximity; and another its proper motion; stars whose proper motions are most conspicuous being naturally deemed nearest.



The two stars, whose Parallaxes have been determined most satisfactorily, are α Centauri, and 61 Cygni. The

Parallax of the former of these was determined by Mr Henderson at the Cape of Good Hope, and that of the latter by Bessel. We will now explain the general effect on a star's place produced by Parallax.

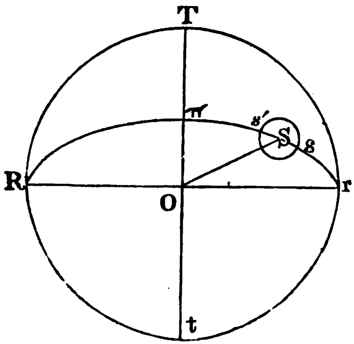
161. *Effect of annual parallax on the places of stars.*

Let S be a star; O the Sun; T any position of the Earth in the ecliptic. Then the angle TSO (fig. Art. 160) is called the *annual Parallax* of the star at the time when the Earth is at T. Now,

$$\sin TSO = \frac{TO}{TS} \sin TOS = \frac{TO}{OS} \sin TOS$$

very nearly; hence the angle TSO is greatest when $\sin TOS$ is greatest, that is, when TOS is a right angle.

Let now the plane of the paper represent the plane of



the ecliptic; and let a plane $R\pi r$ be drawn through the star and the pole of the ecliptic. Then if Tt be a diameter of the ecliptic perpendicular to Rr, it is perpendicular to the plane $R\pi r$, and therefore to OS. Hence the annual Parallax of the star is greatest in the plane through Tt and S, or when the Earth is at T or t. Its position as seen from the Earth will describe a curve about its position S as seen from the Sun, its distance from S being greatest in the direction perpendicular to the plane $R\pi r$, and least in that plane. The effect of Parallax is, then, to make a star describe in the course of a year a small curve about

a mean position; and if its places be compared at intervals of half a year they will be found to differ, the difference being greatest when it is observed from positions of the Earth 90⁰ distant from the plane through the star and the pole of the ecliptic. The effect is similar to that of aberration, except in the fact that the maximum effect happens for each star in that position of the Earth in its orbit for which the effect of aberration is a minimum, and *vice versa* (Art. 142).

162. *Parallaxes of a Centauri and* 61 *Cygni.*

Henderson, by *direct* observations of the bright star *a* Centauri, after applying the corrections, including that for proper motion, detected displacements similar to those described above, and found its greatest Parallax to be 0·98″. This corresponds to a distance from the Sun of more than 206 thousand times the distance of the Earth from the Sun.

The star 61 Cygni, though only of the 5th magnitude, was observed for Parallax, on account of its being known to have a large proper motion. The method adopted was to observe its distances from two stars within a few minutes distance of it, by means of the heliometer. By these *differential* observations all the sources of error, except that incidental to the proper motion of the star, were almost entirely eliminated. From the apparent annual motion of the star as so determined, Bessel assigned an annual Parallax of 0·35″. It has since been observed with more perfect instruments by different astronomers, who agree in assigning a somewhat larger Parallax of about 0·54″.

CHAPTER VIII.

ON THE PLANETS.

163. *The Planets. Bode's law.*

Before the present century, the number of planets known, including the Earth, was seven only; viz. Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and Uranus; Mercury being the nearest to the Sun, and the rest more remote in the order in which they are written.

The orbits which the planets describe about the Sun, as has been explained in Art. 26, are ellipses of small excentricity with the Sun in one of the foci of each ellipse, described in planes making small angles with the ecliptic.

The mean distances of the planets from the Sun were observed to obey pretty closely an empirical law, called Bode's law. This law is, that, if the distance of Mercury be called 4, that of Venus is $4+3$ or 7; of the Earth $4+2\times 3$ or 10; of Mars $4+2^2\times 3$ or 16; of Jupiter $4+2^4\times 3$ or 52; of Saturn $4+2^5\times 3$ or 100; and of Uranus $4+2^6\times 3$ or 196. At about the distance $4+2^3\times 3$ or 28, being the term of the series between the terms which correspond to the distances of Mars and Jupiter, a large number of very small planets have been discovered from time to time since the beginning of the present century; the number at present known amounts to more than eighty.

164. *Perturbations. Discovery of Neptune.*

The orbits of all the planets would be *accurately* ellipses if the Sun were the only attracting body. Since, however, each planet has an attractive power proportional

to its mass and inversely as the square of the distance, the Sun and any planet will not only attract each other, but each will be acted upon by every other body in the solar system.

The effect of any body on the motion of a planet relatively to the Sun depends on the difference of its attractions due to the difference in the distances from it of the planet and the Sun.

Hence arise *perturbations* in the motions of the planets, that is, deviations from the positions they would occupy with respect to the Sun, if the Sun were the only body which attracted them. These perturbations are small; the orbit of any planet therefore does not differ very much from an ellipse; and the corrections to be applied on account of this deviation are small.

The effect of any planet on the motion of any other can be calculated, if the orbit of the former be known with sufficient accuracy.

If the motion of any planet be calculated, taking account of the perturbations due to all the rest, the theoretical places assigned to the planet ought to agree with the places found by observation. The places assigned to Uranus from the results of theory were found to differ from the observed places; and it was suggested that the differences might be accounted for by the action of a planet whose orbit was exterior to that of Uranus. Accordingly Professor Adams in this country and M. Le Verrier in France simultaneously endeavoured to account for the perturbations on this supposition, and to determine the orbit and place of a planet which would produce the perturbations with which Uranus was affected. The efforts of these two astronomers resulted in the discovery, made by each independently in the year 1846, of a planet, whose mean distance from the Sun is about 30 times that of the Earth. This is the planet Neptune.

The empirical law of Bode gives the mean distance of Neptune 38·8 times that of the Earth, which is seen to be much too great.

165. *Superior and inferior planets. Conjunction and opposition. Perihelion and aphelion. Nodes.*

The planets, Earth included, all revolve in the same

direction about the Sun. The excentricities of the orbits of the planets and their inclinations to the ecliptic are all small; and it has been shewn by Lagrange that, although they vary from time to time in consequence of the mutual actions of the planets, yet these variations are confined within certain limits. These limits are small: the orbits will therefore never deviate much from circles, and the planes in which they are described will always be inclined at small angles to the ecliptic.

The planets Mercury and Venus, whose orbits are between the Sun and the Earth's orbit, are called *inferior planets;* those whose orbits are exterior to the Earth's are called *superior planets.*

A planet is said to be in *conjunction* or *syzygy* when its longitude, as seen from the Earth, is the same as that of the Sun. Since the inclination of the planet's orbit is small, the latitude is always small; at conjunction therefore the planet will be seen nearly in the direction of the Sun.

An inferior planet may, at conjunction, be either on the same side of the Sun as the Earth, or on the opposite. In the former case, the planet is between the Sun and the Earth and is nearest to the Earth; it is then said to be in *inferior conjunction.* In the latter case, the Sun is between the Earth and the planet, and the planet is farthest from the Earth; it is then said to be in *superior conjunction.*

The superior planets can never be between the Sun and the Earth; they can, therefore, never be in inferior conjunction.

A planet is said to be in *opposition* when its longitude differs from that of the Sun by 180°. The Earth is then between the planet and the Sun.

It is evident that an inferior planet can never be in opposition.

The angle subtended at the observer's eye by the centres of the Sun and a planet is called the *elongation* of the planet. When the angle is 90° the planet is said to be in *quadrature.*

When a planet is at its least distance from the Sun it is said to be in *perihelion;* and when at its greatest distance, it is said to be in *aphelion.* The least and greatest dis-

tances are called respectively the perihelion- and the aphelion-distance. Perihelion and aphelion are evidently at the extremities of the major axis of the planet's orbit.

The points in which a planet's orbit meets the ecliptic, or where the planet is when its latitude is zero, are called the *nodes* of the orbit. The node in which the planet is when it is passing from the south to the north side of the ecliptic is called the *ascending node;* and the other node is called the *descending node.*

The place which a planet would appear to occupy in the heavens if seen from the Earth's centre is called its *Geocentric* place: and the place which it would occupy if seen from the Sun's centre is called its *Heliocentric* place.

166. *Phases. Cusps.*

The planets are opaque nearly spherical bodies, deriving their light from the Sun. The illumined portion of a planet's surface, at any time, is therefore the hemisphere which is turned towards the Sun, and separated from the unillumined hemisphere by a plane perpendicular to the line joining the centres of the Sun and planet. When the planet is directly between the Sun and the Earth it presents its unillumined hemisphere to the Earth; this can only occur, of course, in the case of an inferior planet. Thus Mercury or Venus, when it is so near its node at inferior conjunction as to be directly between the observer's position and some point on the Sun's disc, appears projected on the disc as a dark spot (Art. 150). When a planet is in opposition or superior conjunction, the whole of its illumined hemisphere is turned towards the Earth. In intermediate positions only part of its illumined hemisphere is visible. The proportion of the visible hemisphere which is illumined is called the *phase.*

When the phase is one-half, or half the disc is illumined, the planet is said to be *dichotomized.* In this case the line drawn from the observer's position to the planet is evidently perpendicular to the line joining the centres of the Sun and planet.

Since the planets describe orbits about the Sun's centre which are approximately circles, and the orbits of the superior planets are exterior to that of the Earth, any position of the Earth is within the area enclosed by the

circle described by a superior planet; the angle between the geocentric and heliocentric distances of a superior planet is therefore never so great as a right angle. Hence never less than half the disc of a superior planet is illumined. A superior planet therefore never appears dichotomized.

When less than half the disc is illumined the planet is said to be *horned;* and the extreme points of the illumined portion are called the horns or *cusps* of the planet. When more than half the disc is illumined the planet is said to be *gibbous*.

167. *Correction for defective illumination.*

In taking an observation of a planet (or the Moon), when not full, with the Transit-Circle, account must be taken of the position of the boundary of the illuminated and unilluminated portions or of the line of cusps, and also of the magnitude of the phase. The circumference of the disc will be a circle half illumined and half unillumined. The horizontal wire, in determining the zenith-distance of the planet's centre, is made to pass successively through the highest and lowest visible points, i.e. through the highest and lowest points of the illumined portion. If, therefore, the line joining the horns does not coincide with the meridian, one of the two positions of the horizontal wire touches the planet's disc, and the other cuts off an unillumined portion. The portion cut off can be calculated from a knowledge of the amount of phase and position of the cusps, or of the position of the planet with respect to the Earth and Sun; the correction made on this account is called the correction for defective illumination. When it has been made, the result of the mean of the observations is the zenith-distance of the planet's centre. In an observation of this kind the two portions of the planet actually observed are called the upper and lower *limbs*.

The difference between the readings for the limbs, after the correction has been made, gives the diameter of the planet.

If the planet's diameter is known, to the reading for the limb observed—corrected if the limb is defective—must be added or subtracted the reading for the semidiameter, by which means the reading for the centre is obtained.

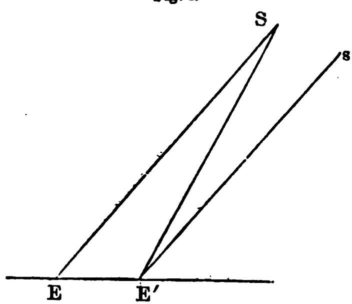
The same method of proceeding applies in taking a *transit* of a planet.

168. *Apparent motions of planets; when direct and when retrograde.*

We now proceed to the explanation of the apparent motions of the planets.

The apparent motion of a planet is said to be direct when it is in the same direction as that of the Sun, or from west to east, or in the order of the signs of the Zodiac. Let E, E' be two positions of the Earth in its orbit, S the Sun. Join ES, $E'S$; and draw $E's$ parallel to ES.

Fig. 1.



Then when the Earth was at E the Sun was seen in direction ES, which is parallel to $E's$: and when the Earth has arrived at E' the Sun is seen in direction $E'S$.

To an observer on the Earth the Sun will therefore seem to have changed its direction from $E's$ to $E'S$.

Again, let P, P' (fig. 2) be the positions of a planet when the Earth is at E, E'. Draw $E'p$ parallel to EP. The planet will appear to have changed its direction from $E'p$ to $E'P'$.

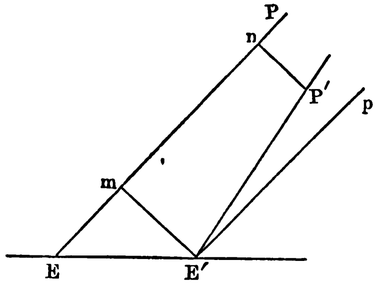
Draw $E'm$, $P'n$ perpendicular to EP.

The apparent motion of the planet is direct if P' in the figure be to the left of p; i.e. if $P'n$ be $< E'm$: also $P'n$, $E'm$ are the spaces perpendicular to EP, described by the planet and the Earth in the same time.

Hence, when EE' is taken indefinitely small, they are

proportional to the velocities of the planet and Earth perpendicular to the line joining them.

Fig. 2.



If, then, the velocity of the planet so resolved be in the same direction as that of the Earth and less, the apparent motion of the planet is *direct.* If it be in the same direction and greater, the apparent motion of the planet is similarly seen to be *retrograde.* And if the velocities resolved perpendicular to EP are equal, $E'P'$ coincides with $E'p$, and the planet appears *stationary.*

If the velocity of the planet perpendicular to EP be in the opposite direction to that of the Earth, P moves to the left of EP, and therefore $E'P'$ is to the left of $E'p$, and the motion is *direct.*

The apparent motion is therefore *retrograde*, only when the resolved velocity of the planet perpendicular to EP is in the same direction as that of the Earth and greater.

169. *Relation between velocity and radius of the orbit.*

By Kepler's third law, the squares of the periods of the planets are as the cubes of the mean distances.

Let then P be the period, v the velocity, a the mean distance of any planet; then since the orbit is approximately a circle with the Sun in the centre, and radius equal to the mean distance,

$$2\pi a = vP\,;$$

$$\therefore a^2 \propto v^2 P^2 \propto v^2 a^3;$$

$$\therefore v^2 \propto \frac{1}{a}:$$

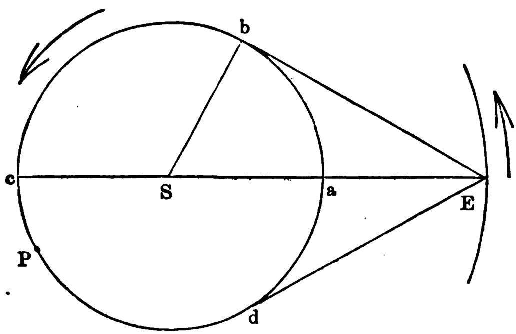
thus, the greater the radius of the orbit, the less the velocity.

It will suffice for the purposes of general explanation, to consider the orbits to be circles, and to be all described in the plane of the ecliptic.

170. *Apparent motions of inferior planets.*

We will first consider the case of an inferior planet.

Let S be the Sun, E the Earth, $abcd$ the orbit of an



inferior planet. Let ES meet the orbit in a and c. Draw the tangents Eb, Ed.

Then the angles bES, dES are equal, and are evidently the greatest angles which can be subtended at E by the planet and the Sun; that is, they are equal to the angle of greatest elongation of the planet.

Suppose now the planet to be at a certain time in inferior conjunction; we may then suppose the planet and Earth to be at a and E.

The velocities of the planet and of the Earth are then perpendicular to Ea, and in the same direction. Also by what has been shewn, the velocity of the planet is greater than that of the Earth. The motion of the planet is therefore *retrograde.*

Again, suppose the planet to have moved from inferior conjunction to its greatest elongation; we may use the same figure, and suppose that the planet is at b when the Earth is at E. Then the resolved velocity of the planet perpendicular to Eb is zero; the apparent motion is therefore *direct*.

Hence, the apparent motion of the planet must be *stationary* at some moment between inferior conjunction and greatest elongation.

Again, at any time when the planet is moving from greatest elongation through superior conjunction to greatest elongation, the planet, Earth, and Sun, are in the relative positions of P, E, and S in the figure, where P is some point between b and c or c and d; and the resolved velocity of the planet perpendicular to EP is evidently in the opposite direction to that of the Earth; the apparent motion is therefore direct during all the time in which the planet is moving from greatest elongation through superior conjunction to greatest elongation on the other side of the Sun.

When the planet is at this second greatest elongation, it is moving towards inferior conjunction, and for the same reason as when the planet was at the first greatest elongation its motion is then direct; and it is retrograde at inferior conjunction; it is therefore stationary at some point between these two positions.

The planet's apparent motion has thus been shewn to be retrograde at inferior conjunction, and for an interval on each side of that position, becoming stationary at two points when it is between greatest elongation and inferior conjunction, and direct through the rest of its orbit.

The figure which we have used serves to illustrate *any particular* configuration of the Earth, Sun, and planet; but it must be remembered, that, by the motion of E, the positions of the points a, b, c, d relatively to E are continually shifting. Thus, between inferior conjunction and greatest elongation, the planet has moved through a space greater than ab. For suppose the planet and Earth to be simultaneously at a and E; when the planet has arrived at b, E has moved in the direction of the arrow, and, not till the planet has moved some distance beyond b, is the

line joining the Earth and planet a tangent to the planet's orbit.

Thus by the motion of the Earth the interval between inferior conjunction and greatest elongation is increased, and the planet in the interval describes a greater arc of its orbit than if the Earth were at rest. And similarly, the interval between either greatest elongation and either conjunction is increased by the motion of the Earth.

171. *Sidereal period of an inferior planet deduced from the synodic period.*

The interval of sidereal time between successive conjunctions of the same sort is called the ***synodic period.***

The synodic period may be expressed in terms of the sidereal periods of revolution of the planet and Earth about the Sun. For let these be p and P, expressed in sidereal days.

Then $\frac{360^0}{P}$ is the angle described by the Earth about the Sun in one day.

And $\frac{360^0}{p}$ is the angle described by the planet in a day.

Therefore $\frac{360^0}{p} - \frac{360^0}{P}$ is the angle by which the radii of the Earth's and planet's orbits separate in a day.

If therefore S be the synodic period, since in S days the radii separate by 360^0, we have

$$\frac{360^0}{S} = \frac{360^0}{p} - \frac{360^0}{P};$$

$$\therefore \frac{1}{S} = \frac{1}{p} - \frac{1}{P};$$

$$\therefore \frac{1}{p} = \frac{1}{P} + \frac{1}{S}.$$

In each mean solar day the Earth has rotated through 360^0 + the angle described by the Sun in its apparent orbit; and this angle amounts in a year of $365\frac{1}{4}$ mean solar days to 360^0 or to a complete rotation of the Earth. Thus in $365\frac{1}{4}$ mean solar days the Earth rotates $366\frac{1}{4}$ times, that is, there are $366\frac{1}{4}$ sidereal days. Putting, then, for P,

$366\frac{1}{4}$, and for S the observed synodic period of Venus or Mercury, we can calculate the sidereal period. For Venus, $S = 584$ sidereal days; whence we get p, the sidereal period of Venus, $= 225$ days, nearly.

The synodic period is of course greater than the sidereal; and this appears from the equation: for, since $\frac{1}{S}$ is less than $\frac{1}{p}$, S is greater than p.

172. *Interval of time between conjunction and greatest elongation.*

If the planet's heliocentric radius has separated A° from the Earth's, the time elapsed $= \frac{A}{360} S$; thus the time in which the planet separates by any angle from the Earth is the same proportion of the synodic period, as the time of separating by the same amount would be of the sidereal period if the Earth were at rest. Hence the interval between inferior conjunction and greatest elongation is increased by the motion of the Earth, in the ratio of the synodic period of the planet to the sidereal period: and similarly for the interval from either conjunction to either greatest elongation.

For example, the angle of greatest elongation for Venus is about 45°. Thus the angle bES, in the figure, $= 45^\circ$; therefore $bSE = 45^\circ$. Hence the interval of time between inferior conjunction and succeeding greatest elongation is about one-eighth of the synodic period,

173. *Kepler's third law confirmed by observation, for the inferior planets.*

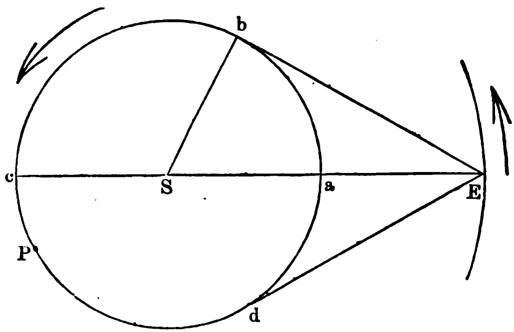
From the angle of greatest elongation the ratio of the mean distances of the planet and the Earth can be found.

Thus the ratio of the mean distance of Venus to the mean distance of the Earth is

$$\frac{Sb}{SE} = \sin 45^\circ = \frac{1}{\sqrt{2}}.$$

If we compare the ratios of the sidereal periods of the

Earth and Venus (Art. 171), we shall find that the result is nearly the same as the ratio of $\sqrt[4]{8} : 1$ or of the square root of the cubes of the mean distances, as it should be according to Kepler's third law.



The interval which elapses between inferior conjunction and greatest elongation

$$=\frac{S}{8}=73 \text{ sidereal days;}$$

the interval which would have elapsed, had the Earth been stationary,

$$=\frac{p}{8}=\text{about } 28 \text{ days.}$$

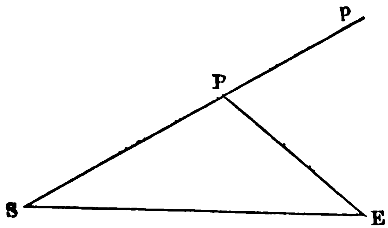
Again, in 28 days Venus describes an eighth part of its orbit about the Sun; therefore in 73 days it describes $\frac{73}{28} \times \frac{1}{8}$ parts $= \cdot 326$ nearly, or nearly a third of the orbit.

174. *Angle of elongation from the Sun at which an inferior planet is stationary.*

To determine the elongation from the Sun at which an inferior planet is stationary.

Let P be the position of the planet, E and S of the Earth and Sun; produce SP indefinitely to p; and let $\angle PES=\theta$, $\angle pPE=\theta'$, $SE=a$, $SP=a'$; and v, v' the velocities of E and P.

Then, the resolved parts of v and v' perpendicular to



PE, are $v \cos\theta$, $v' \cos\theta'$; we must therefore have, when the planet is stationary,

$$v \cos\theta = v' \cos\theta'.$$

Again, $$\frac{a}{a} = \frac{SE}{SP} = \frac{\sin\theta'}{\sin\theta};$$

$$\therefore a \sin\theta = a' \sin\theta' \text{.......................}(1).$$

Also, $$\frac{v^2}{v'^2} = \frac{a'}{a};$$

$$\therefore a' \cos^2\theta = a \cos^2\theta' \text{.......................}(2);$$

therefore eliminating θ' between (1) and (2),

$$a'^3 \cos^2\theta + a^3 \sin^2\theta = a'^2 a;$$

$$\therefore \tan^2\theta = \frac{a'^2 a - a'^3}{a^3 - a'^2 a}$$

$$= \frac{a'^2}{a^2 + aa'};$$

$$\therefore \cot^2\theta = \frac{a^2}{a'^2} + \frac{a}{a'};$$

which gives the angle of elongation at the stationary points, when the ratio of the mean distance of the planet to the mean distance of the Earth is known.

If θ be calculated for Venus it will be found to be an angle of about 26°.

175. *Apparent motion of Venus through a synodic revolution.*

The apparent motion of Venus through a synodic revolution may be described as follows.

At inferior conjunction its motion among the stars is from east to west: it continues in this direction till its

elongation is about 26°. It then appears for a time to be stationary, and then returns in the direction of the Sun's motion, or from west to east. For some time after Venus has reached the stationary point, the Sun gains on the planet in direct motion until the elongation is about 45°. The motion of Venus still continues direct, and exceeds that of the Sun, passing it at superior conjunction, and getting more and more to the east of the Sun till its elongation is about 45°. The motion still continues direct, but the rate becoming less than the Sun's, the Sun begins to approach it: when the elongation has thus become diminished to about 26°, Venus becomes stationary again, and then retrograde, passing with retrograde motion to inferior conjunction; and from that point the whole series of apparent motions commences again in the same order, the motion continuing retrograde till the elongation is 26°, and so on.

While this series of apparent motion has been going on, Venus will have made two revolutions in its actual orbit, and more than half completed a third. The interval of time elapsed has been 584 days, in which the Sun has described more than a revolution and a half from west to east. This is therefore the amount by which the direct motion of Venus has gained on the retrograde.

A similar description applies to the apparent motions of Mercury. The distance of Mercury from the Sun being much less than that of Venus, the angle of greatest elongation is much less, being about 22° or 23°; also the sidereal period is only about 88 days, and the synodic period 116 days nearly.

When Venus is to the west of the Sun, it rises before the Sun; it is then called the *morning star;* when it is to the east of the Sun, it sets after the Sun, and is then called the *evening star*. The brightness of Venus varies in different parts of its orbit from two causes; one is the phase of the planet, and the other its distance from the Earth. Its brightness is greatest when it is at an elongation of about 40° from the Sun in the superior part of its orbit.

176. *Apparent motion of a superior planet.*

The orbits of the *superior* planets being exterior to that of the Earth, the elongation may evidently be any

whatever. They are not, therefore, like the inferior planets, confined in their apparent motions within a certain angular distance from the Sun.

When an *inferior* planet appears stationary the line joining it with the Earth is for the moment moving parallel to itself: the Earth would therefore appear stationary to an observer on the inferior planet. So to an observer on the Earth a superior planet appears stationary at those points at which the Earth appears stationary as seen from the superior planet.

Hence a *superior* planet appears stationary at two points, before and after *opposition;* and at opposition, since the Earth is moving faster than the planet the apparent motion is opposite in direction to that of the Sun; it is therefore retrograde at opposition. And in moving from opposition to the succeeding stationary point, the retrograde motion diminishes to zero, and the motion becomes direct till the planet arrives at the stationary point which precedes opposition, when it becomes stationary again, then becomes retrograde and continues so through opposition to the stationary point which succeeds opposition.

Again, the time between two consecutive oppositions is equal to the synodic period of the planet; and the angle which the planet describes about the Sun in that time is the angle between the positions of the line joining the planet and the Earth at the two oppositions. And since the planet's motion in its orbit is direct, it is the angle *gained* by the direct over the retrograde motion between two consecutive oppositions, that is, in a synodic period.

Hence a superior planet's motion is retrograde before and after conjunction, and direct in the remainder of the synodic period; and the direct motion is in excess of the retrograde. Also the apparent direct motion of the planet between two oppositions is the actual angular motion of the planet about the Sun during the interval. Thus by comparing observations of a superior planet at oppositions separated by long intervals, the mean angular velocity in its orbit, or the *mean motion* as it is called, may be found.

If P, p be the sidereal periods of the Earth and a superior planet, S the synodic period; then

$$\frac{360^\circ}{S} = \frac{360^\circ}{P} - \frac{360^\circ}{p}:$$

$$\therefore \frac{1}{S} = \frac{1}{P} - \frac{1}{p}:$$

whence S is greater than P, or the synodic period of a superior planet is greater than a year, as may be seen otherwise. This equation determines the sidereal period when the synodic period has been found by observation.

177. *Rotation and ellipticity of planets.*

Those planets which are sufficiently large and near the Earth for their surfaces to be accurately observed, have been found, by observing the motions of certain spots on their surface, to rotate about an axis; the period of rotation, and the inclination of the axis of each planet to its orbit, being different for different planets. Mars, Jupiter, and Saturn, when observed with powerful telescopes, are found to be not quite spherical, but spheroidal, the least diameter coinciding with the axis of rotation of the planet; thus agreeing exactly with what we should expect from the analogy of our own Earth. Saturn is accompanied by a ring, or rather two or three separate rings, in the plane of its equator, and revolving about the centre of the planet.

CHAPTER IX.

THE MOON AND SATELLITES.

178. *The Moon's orbit. Retrograde motion of the nodes. Progressive motion of the line of apsides.*

It has been stated (Chapter I.) that the Moon is the nearest to us of all the heavenly bodies, and revolves in an orbit which is approximately an ellipse of small excentricity with the centre of gravity of the Earth and Moon in one focus. Its mean distance from the Earth is about 240,000 miles, being about 60 times the Earth's radius. The actual motion of the Moon in space results from the combination of the motion of the Earth about the Sun and of the Moon about the Earth. The actual curve described by it is concave to the Sun in every part.

The apparent motion of the Moon in the heavens is direct, i.e. from west to east; it is therefore in the same direction as that of the Sun; but it is much more rapid, its sidereal period being about $27\frac{1}{3}$ days; thus it completes 13 revolutions about the Earth before the Earth has completed one about the Sun.

The plane of its orbit is not coincident with the ecliptic, but is inclined to it at an angle of rather more than 5^0. The points of intersection of the orbit with the ecliptic, or the nodes, have a rapid retrograde motion among the stars, the period of a revolution of the nodes being about $18\frac{1}{2}$ years: thus in one year the nodes retreat through about $19\frac{1}{2}^0$, and in a day through more than $3'$. The inclination

of the plane of the orbit is in the meantime subject to periodic fluctuations, but never differs much from 5^0. The general nature of the motion of the Moon about the Earth may therefore be conceived by supposing it to move in an ellipse, the plane of which moves so that its inclination to the ecliptic is pretty constant, while its line of intersection regredes on the ecliptic at the rate of about $3'$ a day.

The line of apsides, or major axis of the Moon's ellipse, is also in rapid motion; the motion being direct, and at such a rate as to complete a revolution in about 9½ years.

These irregularities in the Moon's motion are due to the disturbing attraction of the Sun. Besides the above motions, which are necessary to be stated in order to give a notion of the general nature of the Moon's apparent path in the heavens, there are many others of great importance, due to the same cause. For a discussion of these *inequalities* of the Moon's motion, as they are called, the student is referred to more complete treatises.

179. *Moon's mean synodic period found by means of ancient eclipses.*

When the Moon is in conjunction, if its node is within a certain distance of the Sun, the Moon comes between the Earth and Sun, and appears to cover, partially or entirely, the Sun's disc; when this happens there is an eclipse of the Sun (Chapter XI.). The interval between successive conjunctions is called a synodic period of the Moon, or a lunation; it is greater than the sidereal period, and is about 29½ days; the difference being due to the direct motion of the Sun. The interval between any two eclipses is an integral number of synodic periods; if, therefore, the times of two eclipses be known, the interval between them is known. In the case of recent eclipses, the number of lunations between the eclipses is known; the interval between two eclipses divided by this number gives the mean of the synodic periods for that interval. By this means a very accurate value of the mean synodic period is found. This value can then be used in calculating backwards the particular conjunctions at which eclipses must have happened and determining the precise times at which they occurred. By this means eclipses recorded in very remote times can

be identified; and the number of lunations in the interval between an ancient and a modern eclipse, in such a case, becomes known. If the interval between two such eclipses be divided by the number of lunations, the mean of the synodic periods which have occurred between them becomes known. In this way, from an eclipse recorded to have happened at Babylon in the year 721 B.C., March 19, and other ancient eclipses, the mean synodic period of the Moon has been found to be about 29·53 days.

180. *The sidereal period deduced from the synodic.*

The synodic period being known, it is easy to deduce the sidereal period. For, let P be the synodic period, p the required sidereal period, and let T be a sidereal year, all expressed in mean solar days.

Then $\frac{360^0}{p}$ is the number of degrees by which a declination-circle through the Moon has separated from a fixed declination-circle in a day: also $\frac{360^0}{P}$ is the number of degrees by which a declination-circle through the Moon has separated from one through the Sun in a day: and $\frac{360^0}{T}$ is the number of degrees by which a declination-circle through the Sun has separated from a fixed one in a day.

Thus,
$$\frac{360^0}{p} = \frac{360^0}{P} + \frac{360^0}{T};$$
$$\therefore \frac{1}{p} = \frac{1}{P} + \frac{1}{T};$$
or
$$\frac{1}{p} = \frac{1}{29{\cdot}53} + \frac{1}{365{\cdot}256};$$
whence p will be found to be 27·32 days.

181. *Period of rotation of the Moon equal to the sidereal period of revolution in its orbit.*

The surface of the Moon is seen with the naked eye to be very uneven, and covered with dark spots; this appearance is due to the existence of mountains of considerable height, as is proved by the shadows cast by the Sun on the Moon's visible surface when it is not fully illumined, these

shadows being seen as dark lines on the surface in the direction of the line joining the Moon and Sun.

These spots are observed to maintain almost exactly the same position with regard to the disc, whatever the position of the Moon in its orbit. Hence a line joining the centres of the Earth and Moon meets the surface of the Moon in nearly the same point on the surface in all positions of the Moon; the straight line, therefore, which joins this point with the centre of the Moon is not fixed in direction, but changes its direction in such a manner as to describe an angle of 360^0 in space in a sidereal revolution of the Moon. Hence the Moon rotates about an axis which is nearly perpendicular to its orbit, and completes a rotation in a sidereal period.

182. *Changes of phase of the Moon in the course of a synodic period.*

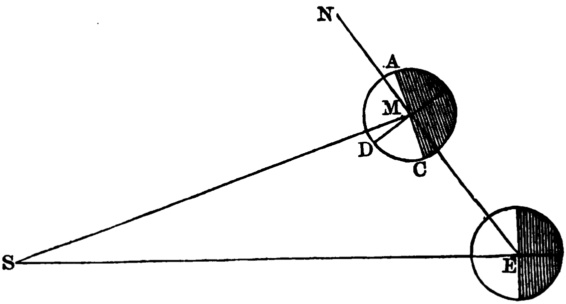
Except at opposition the Moon is never seen with its whole disc illumined; and the proportion of it which is illumined is observed to depend simply on the position of the Moon with reference to the Sun. At conjunction, or new Moon, when the Moon is nearly in the same direction as the Sun, it is overpowered by the Sun's rays, and is not visible at all: as soon as it has moved from the Sun sufficiently far to be distinguished, which will be in two or three days, it is seen as a thin crescent, with its convex boundary turned towards the Sun. As the Moon recedes from the Sun, the width of the crescent increases, till when the Moon is in quadrature half the disc is illumined; during this period the illumined part has been crescent-shaped and the Moon is said to be *horned;* as the Moon proceeds the width of the bright part increases, both boundaries being convex; in this state the Moon is said to be *gibbous*. When the Moon has arrived at opposition, the whole disc is illumined, and it is *full Moon:* from opposition to quadrature the breadth of the illumined part diminishes, the Moon at quadrature being again half-illumined: from quadrature to conjunction the illumined part is crescent-shaped, continually diminishing till it is again lost in the rays of the Sun.

183. *Explanation of the changes of phase.*

We shall now shew how these changes of phase are

explained, by considering the Moon as an opaque body deriving its light from the Sun; since the Moon's orbit is inclined at only 5^0 to the ecliptic, we shall, for simplicity, consider it to coincide with the ecliptic.

Let S, E, M be the positions of the centres of the Sun, Earth, and Moon at any time; draw AC, MD in the plane SEM perpendicular to SM, EM respectively. Then a plane through AC perpendicular to SM intersects the surface of the Moon in the boundary between the illumined and unillumined halves; for on account of the distance of the Sun, the rays from it to the Moon are



nearly parallel to SM. Again, a plane through MD perpendicular to ME separates approximately the half of the Moon which is seen from E from the other half. Hence of the visible portion only the part represented by DC is illumined; the visible illumined surface is therefore proportional to the angle DMC.

Produce EM to N; then

$$\angle DMC = 90^0 - \angle DMS = \angle SMN;$$

also, on account of the great distance of the Sun, the angle ESM is never very great; the angles SMN, SEM are therefore nearly equal; and hence the angle DMC is nearly equal to the angle SEM. Thus as the Moon proceeds from quadrature to quadrature through conjunction, the angle DMC varies from 90^0 through 0^0 to 90^0 again: the Moon is therefore in this part of the orbit crescent-

shaped; and the proportion of the disc which is illumined varies from half through zero to half again; this agrees with the observed phenomena. Similarly the phonomena of the phases in the other half of the orbit may be explained.

When the Moon is full the Sun and Moon are at exactly opposite points of the heavens,—supposing as before that the Moon's orbit coincides with the ecliptic. The Moon is therefore below the horizon during the day and above during the night, and crosses the meridian at midnight; and the Sun and Moon are at the same distance from the equator, on different sides of it. Thus at midwinter, when the Sun has its greatest south declination, the Moon has its greatest north declination; at midwinter, therefore, the full Moon's meridian altitude is greatest; so at midsummer the full Moon's meridian altitude is least; also from the autumnal to the vernal equinox the full Moon's meridian altitude is constantly greater than from the vernal to the autumnal.

When the Moon is new, it is between the Earth and the Sun; the whole of the Earth's surface which is turned towards the Moon is therefore illumined; and when the Moon is full all the dark half of the Earth is turned towards the Moon. It is easily seen that in any other position of the Moon the phases of the Earth and Moon are supplementary to each other, the illumined portion of the Moon's surface visible to the Earth being the same part of its whole surface, as the unillumined portion of the Earth visible from the Moon is of the Earth's whole surface—the small angle subtended by the Moon's orbit at the Sun being neglected.

184. *Sun's distance cannot practically be deduced from observation of the Moon when dichotomized.*

When the Moon is half full, or dichotomized, the plane of separation of its illumined and unillumined parts passes through the Earth. Thus, in the figure, p. 161, AC coincides with ME; therefore $\angle SME$ is a right angle. Hence if $\angle SEM$ the angle of elongation of the Moon $=E$, $ME=SE \cos E$. Hence, if the elongation of the Moon, when dichotomized, be observed, we have an equa-

tion for determining the Sun's distance, that of the Moon being supposed known.

Practically this method is of no value, on account of the impossibility of accurately ascertaining when the Moon is dichotomized; and a small error in determining this would very largely affect the result. For, since ME is very small compared with SE, $\cos E$ is very small, and $\angle E$ very nearly a right angle. Now suppose E to be the true, and $E+h$ the observed elongation, then

$$\cos E - \cos(E+h) = 2\sin\frac{h}{2}\,.\,\sin\left(E+\frac{h}{2}\right);$$

$$\therefore \frac{\cos E - \cos(E+h)}{\cos E} = 2\sin^2\frac{h}{2} + \sin h\,.\,\tan E.$$

Since E is nearly a right angle $\tan E$ is very large; hence $\sin h \tan E$ may be large even when h is small. Hence, the error in the Sun's distance as determined by this method may bear a large ratio to the Sun's distance.

185. *Librations of the Moon.*

It has been shewn that the Moon rotates about an axis nearly perpendicular to the plane of its orbit. The Moon's equator, i.e. the plane through its centre perpendicular to the axis, is therefore nearly coincident with its orbit; it is inclined to the orbit at an angle of about $1\frac{1}{2}^\circ$.

It is probable, from dynamical considerations, that the period of rotation of the Moon, like that of the Earth, is accurately constant: a radius of the Moon's equator therefore describes *equal angles* in equal times. Since, however, the Moon's orbit about the Earth is an ellipse, a line joining the centres of the Earth and Moon describes *equal areas* in equal times; this line will therefore have a variable angular velocity which is greatest at perigee and least at apogee. Since, however, the point in which it meets the surface of the Moon never deviates much in position from a fixed point on the surface, the angular velocity of rotation of the Moon is the mean of the angular velocities of the Moon in its orbit: thus at perigee, the Moon's orbital motion is greater than its motion of rotation, and the surface presented to the observer will vary slightly, so that a small portion becomes visible on that side which it would present if there were no rotation,

i.e. a portion of the west edge; similarly at apogee, the Moon would shew a portion of the east edge; and the amounts of its surface on either side which the Moon presents in this manner are the same at every perigee and apogee. Of course, since the excentricity of the elliptic orbit is very small, the portions of surface thus shewn are very small.

This phenomenon is called the *Libration in Longitude.*

Again, since the axis of rotation of the Moon is not perpendicular to its orbit, its inclination to the radius of the orbit will vary in different parts of the orbit; thus, at different times either pole of the Moon is differently presented to us; and different parts of the surface about the poles are presented to us. Since the Moon's axis is so nearly perpendicular to the orbit, the surfaces thus alternately presented and hidden are very small.

This is called the *Libration in Latitude.*

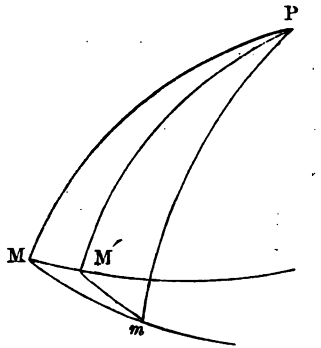
Lastly, an observer at any place on the Earth will, on account of parallax, see more of the Moon's upper limb when the Moon is on the horizon, and more of its lower when the Moon is high up; and the variations in the surface shewn from this cause will be greater the greater the meridian altitude of the Moon. Also, the upper and lower limb will be different for observers in different latitudes. Thus there will for each latitude be a daily variation due to parallax, the variation depending on the latitude of the observer.

This is called the *Diurnal Libration.*

186. *Harvest Moon.*

In consequence of the orbital motion of the Moon from west to east, it rises later every day. If the orbit of the Moon coincided with the equator, and the angular velocity about the Earth were uniform, the time by which the Moon's rising is retarded would be the same from day to day; but since the Moon's orbit is inclined to the horizon at different angles at different times, the amount of retardation varies considerably. We propose here to consider the retardation of the Moon at different times of the year, the Moon's orbit being supposed to coincide with the ecliptic.

Let P be the north pole, M the Moon at rising on any day, Mm the arc of its orbit described in a day, mM' an arc of a small circle parallel to the equator, meeting the horizon in M'; then M' is the position of the Moon at rising the next day; for when the point M of the orbit meets the horizon the next day the Moon is at m below the horizon, and its diurnal circle is mM'. Now Mm is nearly constant, and the small circle mM' is inclined to the horizon at an angle equal to the co-latitude; thus mM' is clearly least when the angle mMM' is least, or when the Moon's orbit is inclined at the least angle to the horizon; and greatest, when it is inclined at the greatest angle.



Again, for a given value of $M'm$, the angle $M'Pm$ is easily seen to be least when Pm is 90^0; the retardation in the time of rising is therefore least when the Moon is crossing the equator, and at the time when the Moon's orbit is least inclined to the horizon. Now the diurnal circle of the pole of the ecliptic is a small circle about P; its angular distance from the zenith is therefore greatest when it crosses the meridian below P, and least when it crosses it above P; thus, when the equator and ecliptic intersect on the horizon, and the part of the ecliptic below the horizon is between the equator and horizon, the inclination of the ecliptic to the horizon is least.

Hence the retardation is least when the Moon is in Aries.

Thus when the Moon at full is nearest to Aries the retardation is less than for full Moons in all other positions; but in this case the Sun is in Libra. Thus, of all full Moons in a year, that which happens nearest the time of the Autumnal equinox, or nearest the 22nd of September, is the least retarded in its time of rising on successive

evenings. Its time of rising is therefore for several consecutive evenings nearly coincident with the time of sunset. The full Moon which happens about this time is called the *Harvest Moon.*

If the inclination of the Moon's orbit to the ecliptic be taken into account, the retardation is easily seen to be least of all when the Moon and the ascending node of its orbit are at Aries together.

In our latitudes the retardation may be as small as 15 minutes, and as great as 75.

187. *Phenomena visible from the Moon.*

It will be instructive to consider what would be the general nature of the appearances of the heavens to an observer on the Moon.

Since the Moon rotates on its axis once a month, the heavens will appear to revolve in the same period, the diurnal circle of any star being parallel to the Moon's equator. Again, since the orbital velocity of the Earth about the Sun is greater than that of the Moon about the Earth, the general direction of motion of the Moon in space is the same as that of the Earth; also, as has been said, the Moon's orbit is everywhere concave to the Sun. Thus the Sun will appear to have a motion relative to the stars; and this motion will be in the opposite direction to the motion of the heavens due to the Moon's rotation; for the general direction of the Moon's rotation is the same as that of its motion about the Sun. The apparent motion of the Sun will be most rapid when the Earth is between the Moon and the Sun, and least in the rest of the month or of the Moon's day.

The Earth will be seen by observers on one half of the Moon in a fixed direction—neglecting the librations—while the rest of the heavens revolve, and will be observed to go through all its phases in one of the Moon's days: observers on the other half never see the Earth at all. To an observer on the hemisphere facing us the Earth will be *full* when the Sun is below his horizon at the opposite part of the heaven to the Earth; the Earth will thus illuminate the Moon more or less during the Moon's night: this illumination of the Moon by the Earth explains why for a short

time before and after new Moon the part of the Moon unillumined by the Sun is visible to us. The Earth will be *new* when the Sun is seen nearly in the same direction as the Earth.

188. *Moon's atmosphere, if any, must be extremely rare.*

The Moon either has no atmosphere, or, if it has one, it must be at least 1000 times rarer than that of the Earth. This is proved principally by observations of stars when the Moon in its orbit passes between them and the Earth, so as to *occult* them; when this happens the light of the star, which comes to the eye of an observer on the Earth just before and after the occultation, passes through the strata of the Moon's atmosphere—if there is one—in a direction almost tangential to the strata; the refraction is therefore the greatest possible. The light of the star would therefore be seen for some time after the contact of the Moon with the star at the beginning of the occultation, and before the actual emergence of the star from behind the Moon at the end; the period of an occultation would, on both accounts, be less than the calculated time of describing the chord of the Moon joining the points of first contact and emergence. But no difference has been observed between the calculated and observed times large enough to be accounted for by a lunar atmosphere which is not of extremely small density.

Besides the effect which an atmosphere would have in shortening the duration of an occultation, the brightness of the star would be perceptibly diminished by it just before and after the occultation; this effect, however, is not observed.

It remains to mention another effect which would necessarily follow from the existence of a lunar atmosphere. It is that in consequence of twilight rather more than half the Moon's surface would be illumined by the Sun. Hence, when the Moon is a thin crescent, and the separation of the illumined from the unillumined parts consequently very clearly defined, the illuminated edge of the Moon should be more than a semicircle. This appearance has been observed, but to such a small extent as to indicate an atmosphere of extreme tenuity.

189. *Satellites.*

Jupiter is attended by four satellites, three of which move in nearly circular orbits about the centre of their primary, their orbits being nearly in the plane of his equator; the orbit of the fourth is an ellipse of considerable excentricity, and its plane is inclined to the equator at about 8^0. A remarkable relation has been observed between the angular velocities of the first three in their orbits about Jupiter; it is that the sum of the angular velocity of the first and twice that of the third is equal to three times that of the second.

Saturn is known to have eight satellites. Of these the most remote from Saturn has been noticed to have a remarkable variation of brightness according to its position in its orbit, the minimum brightness always occurring at a definite elongation from Saturn. It has been concluded from this that the satellite rotates about its axis in the same time as that in which it revolves about its primary, an inference which is supported by the analogy of the case of our Moon.

Uranus has at least four satellites, moving in orbits nearly perpendicular to the ecliptic.

Neptune has one satellite at least, the orbit of which is inclined to the ecliptic at an angle of about 35^0.

CHAPTER X.

ON THE DETERMINATION OF GEOGRAPHICAL LATITUDE AND LONGITUDE.

LATITUDE.

190. *Latitude determined by observations of circumpolar stars.*

The latitude of a place is (Art. 12) the angular distance of the zenith of the place from the celestial equator, measured on the celestial meridian. The complement of this angle, which is the angular distance of the zenith from the pole, is called the co-latitude.

If there existed a star situated exactly at the pole, we should be able at once to find the co-latitude at a fixed Observatory, by observing with the Transit Circle the zenith-distance of that star. There is, however, no such star; we cannot therefore observe the co-latitude directly. It may, however, be inferred from observation of a circumpolar star above and below the pole. If the observed zenith-distances of such a star at its superior and inferior transits be corrected for refraction and aberration, half their sum will be the zenith-distance of the pole.

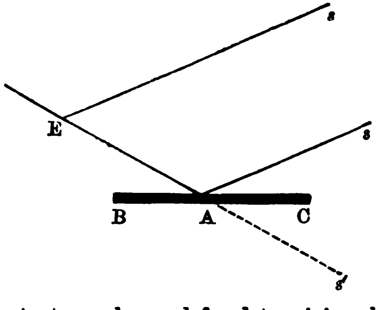
In practice, after the co-latitude has been determined approximately by this, or any other method, it is applied to determine the N.P.D.s of a number of circumpolar stars from their zenith-distances both at their upper and lower

transits: each star being observed several times at each transit, and the mean of the observed Z.D.s at the upper transit taken as the correct Z.D., and similarly for the lower transit.

If the assumed co-latitude be correct, the N.P.D. as determined from the upper transits of any star will be equal to that found from the lower transits. Suppose, however, the co-latitude assumed be too great; the upper transits will place the pole too low by the amount of the error, the lower transits too high by the same amount. The N.P.D.s will therefore differ by twice the amount of the error of the assumed co-latitude: the same may similarly be shewn to be true if the assumed co-latitude be too small. If, then, half the difference of the N.P.D.s, as determined by upper and lower transits of any star, be taken, and the mean of the results for several stars found, we shall have a very correct determination of the error of the assumed co-latitude: the latitude will therefore be very accurately determined.

191. *Latitude determined by observations with a sextant.*

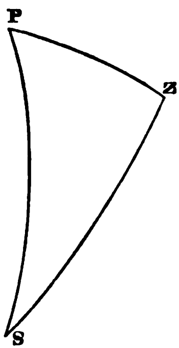
Where there is no fixed observatory, and consequently no meridian-instruments, the co-latitude may be determined by observations with a Hadley's sextant.



The sextant may be used for determining altitudes by taking the angle which a star and its image subtend at the

observer's eye. Thus, let *s* be a star, *s'* its image reflected from a trough of mercury; *BC* the surface of the mercury; *sA* a ray reflected in the direction *AE*; *E* the observer's eye; *Es*, parallel to *As*, the direction of the star. Then, the angle *sEA* is equal to the angle *sAs'*, or to twice the altitude of the star. Now, by means of the sextant an observer can determine the angle subtended by *s* and *s'* at his eye: and this angle, by what has preceded, is twice the star's altitude. Hence the altitude of the star is known.

If, now, the star be on the meridian at the time of observation, the zenith-distance of the star is the sum, or difference, of its N.P.D. and of the co-latitude of the place; thus the altitude of a known star would determine the co-latitude. In the absence of a meridian instrument, however, this cannot be secured; and we must observe the star twice, once before, and once after passing the meridian, when its altitude is the same as it was at the first observation. The star is, at the second observation, precisely at the same distance from the meridian as at the first, and its hour-angle is the same. Hence the difference between the sidereal times of the observation gives twice the hour-angle at either observation. Hence, if *P* be the pole, *Z* the zenith, *S* the star, the observations give *ZS* and the angle *ZPS*; and the Nautical Almanac gives *SP*, if the star is known. These three data quite determine the spherical triangle *SPZ*: hence *ZP* the co-latitude can be found.



The same method is applicable to the Sun; but in this case the change of N.P.D. in the interval between the observations must be taken into account in the calculation

LONGITUDE.

192. *Methods of determining the longitude of a place.*

It has been explained in Chapter v. that both the sidereal and the mean solar time differ for places in different longitudes at the same instant of absolute time; the reason being that the commencement of a day is reckoned

at each place from the time of transit of the first point of Aries or the mean Sun over the *meridian of the place.*

Since the declination-circle through the mean Sun separates, from any given meridian of the Earth, by equal angles in equal times, the difference of longitude of any two places is proportional to the interval elapsed between the times of transit of the mean Sun over their meridians, i.e. to the time indicated by a mean solar clock at the first meridian at the instant when the mean Sun is on the second; thus, the difference in the times of a phenomenon occurring at a given instant as recorded by clocks at two places is proportional to their difference of longitude. This is the principle of all the methods used for determining longitudes.

We shall briefly explain the most important methods for this purpose; these are

(1) By terrestrial signals observed simultaneously at the two places.

(2) By transmission of chronometers.

(3) By eclipses of Jupiter's satellites.

(4) By Moon-culminating stars.

(5) By the Moon's greatest altitude.

(6) By lunar distances.

(7) By eclipses of the Moon and Sun, and by occultations of stars by the Moon.

193. (1) *The method by signals.*

A rocket or a mass of gunpowder is exploded at a place intermediate between the two stations; the difference of the local times of the phenomenon at the instant at which it is observed at the two places gives their difference of longitude.

This supposes the places to be so near to each other that the same signal may be visible to both. If they are too far off, a series of intermediate stations are selected so near to each other that observers at any two consecutive stations can observe a signal given at some point between them. By this means the difference of longitude between each pair can be found; and thus the difference of longitude between the stations for which it is required becomes known.

When two Observatories are in telegraphic communication with each other, the time of transit of any star, taken by the galvanic method (Art. 58), at one of the Observatories can, by connecting the galvanic wires of the transit instruments at the two Observatories with the telegraphic wires, be recorded simultaneously at both stations. We thus obtain the local times at the two Observatories of the transit of the star over the meridian of one of them; the difference gives the hour-angle between the meridians or the difference of longitude.

We have assumed the transmission of the galvanic current to be instantaneous. This is not quite true, though the velocity is extremely great; and by taking the mean of the results given by transit at the two Observatories, the error arising from this cause is eliminated. Suppose, for example, that Greenwich and Paris are the two stations, then, since Greenwich mean noon occurs after that of Paris, the clock at Greenwich at any given instant is behind the Paris clock; the time in which the current travels from Greenwich to Paris will, therefore, by causing the observation to be recorded at Paris later than it should be, *increase* the interval between the recorded local times of the transit of the star across the meridian of Greenwich, by the time which the current takes to travel from Greenwich to Paris. Similarly, the difference of local times given by an observation at Paris is as much *diminished*. The mean of the observations, therefore, gives a correct result.

194. (2) *The method by transmission of chronometers.*

The principle of this method is very obvious. If the error of a chronometer and its rate are known, and if its rate can be assumed to be steady, a chronometer transmitted from one place to another indicates at the latter place, at any given instant, the local time of the former: this, compared with the local time at the latter, as corrected by observations of transits, gives, by what has been said, the difference of longitude between the two places. The application of this method is encumbered with practical difficulties, into which we cannot enter here.

195. (3) *The method by eclipses of Jupiter's satellites.*

Since the planets and satellites derive their light from the Sun, it happens sometimes that one of these bodies, being interposed between the Sun and another body, deprives the latter of the light of the Sun, and renders it invisible. It thus happens that, in consequence of the motions of the satellites of Jupiter in their orbit round their primary, they enter at intervals into the shadow caused by the interposition of its body between them and the Sun. Since the disappearance and reappearance of a satellite of Jupiter, on entering into and emerging from the shadow, happen each at a definite instant of absolute time, the difference between the recorded local times at different places at which each of these phenomena happens, affords a means of determining the difference of longitude of the places.

196. (4) *The method by Moon-culminating stars.*

The Moon by its orbital motion about the Earth separates in R.A. from any fixed star by 360^0 in 27·32 days. In one *day*, therefore, its motion in R.A. is rather more than 13^0; at the beginning and end of an hour it transits two meridians nearly 15^0 apart, and in this interval of time its motion in R.A. is more than half a degree.

If, then, the R.A. of the Moon be ascertained at its transit over the meridian of a place on any day, and if the rate of its motion in R.A. at the time be known, and the R.A. which it had at the instant of its preceding transit over the meridian of Greenwich, the difference of the R.A.s at the place and at Greenwich determines the longitude of the place.

For the purpose of facilitating the application of this method the Nautical Almanac gives the R.A. of the Moon's bright limb at each transit over the meridian of Greenwich, the sidereal time occupied by a semi-diameter of the Moon in its transit, and the motion in R.A. of the Moon in the interval of its transits over two places differing by 15^0, or one hour, in longitude. If, then, A^0 be the difference between the observed R.A. of the Moon's centre at its transit over any meridian and the given R.A. at its preceding transit over the meridian of Green-

wich, and R^0 the given motion in R.A. for 15^0 of longitude, we have

$$R^0 : A^0 :: 15^0 : \text{required longitude};$$

from which proportion the longitude of the place can be calculated.

In order to determine the R.A. of the Moon's centre at its transit over the meridian of the place, the difference of sidereal time between the transits of the Moon and a star near it is taken; from which, if the R.A. of the star is known, that of the Moon can be deduced (Art. 33); the star so observed should have as nearly as possible the same N.P.D. as the Moon, in order that the instrumental errors—which depend on the position of the object observed at its transit over the meridian—may be as nearly equal as possible for the Moon and star. The places of certain stars satisfying this condition are recorded in the Nautical Almanac; these stars are called *Moon-culminating stars.*

197. (5) *By the Moon's greatest altitude.*

The Nautical Almanac gives the Moon's declination for every day of the year at the time of its transit over the meridian of Greenwich; if, then, the Moon's declination at its transit over any meridian can be ascertained, it will be found to lie between the declinations at the preceding and succeeding transits at Greenwich. Hence, assuming the rate of change of declination of the Moon between two successive transits to be constant, the Greenwich time, at which the Moon had the declination it was found to have at the place of observation, becomes known; hence if the local time of transit be known, the difference gives the longitude of the place.

The Moon's declination is the difference of its meridian altitude and of the co-latitude of the place; on account of the change of declination the Moon's meridian altitude does not quite coincide with its greatest altitude; if therefore the Moon's greatest altitude be observed, a correction must be applied, depending on the rate of change of the Moon's declination, in order to get the meridian altitude. When this has been done the longitude of the place can be inferred, as has been explained.

198. (6) *By lunar distances.*

This is the method used at sea; the observations being such as can be taken with a sextant. The principle of the method is the same as that of the method by Moon-culminating stars; the determination by observation of the distance of the Moon's centre from a planet or bright star playing the same part in this process which the determination of its R.A. does in the other.

The Nautical Almanac gives, *for every three hours* of mean solar time, the angular distances of the Moon's centre from certain planets and bright stars, corrected for refraction and parallax; from this by proportional parts, or by a process called interpolation, we may deduce with great accuracy the distances of the Moon from the same objects for any intermediate time—the time being of course reckoned from Greenwich mean noon.

If, then, the longitude is required at any place, the distance of the Moon's bright limb from a star or planet is observed with the sextant; this has to be corrected for the angular radius of the Moon, in order to get the distance of the Moon's centre.

The *altitudes* of the Moon and star are also taken, either at the same time, or before and after the lunar distance; if the latter, the times of all the observations being known, the altitudes at the time of taking the lunar distance can be deduced. The places of the Moon and star can now be corrected for parallax and refraction, both of which corrections depend upon the altitude—this is called *clearing* the lunar distance;—the correct geocentric distance of the Moon from the object observed is thus known. Now this geocentric distance will be intermediate between two consecutive lunar distances the Greenwich mean times of which are given in the Nautical Almanac. Thus, the Greenwich mean time of the observation can be deduced; and this, compared with the local time, gives the longitude of the place.

199. (7) *By eclipses of the Sun and Moon, and by occultations of stars by the Moon.*

The entrance of the Moon into, and emergence from the shadow of the Earth thrown by the Sun, happen, as in the

case of Jupiter's satellites, at definite instants of absolute time; and the difference of longitude of two places can therefore be determined by an eclipse of the Moon in the same manner as by an eclipse of a satellite of Jupiter. But this method is practically of little value, on account of the infrequency of eclipses of the Moon, and of the impossibility of determining the precise moments of the beginning and end of one.

An occultation of a star is a phenomenon which can be observed very accurately, and the local time of the phenomenon can therefore be determined with exactness. Also from the tables of the Moon's motion its exact geocentric position at any Greenwich mean solar time can be found, and consequently the times at which an occultation would appear to begin and end to an observer at the centre of the Earth; and hence the Greenwich times of the beginning and end can be calculated for any place on the surface of the Earth, or for any given longitude. Again, for a given longitude the local time corresponding to a given Greenwich time can be found; and thus the local times for an occultation at any given place can be calculated. If, then, an occultation be observed at any place, and the local time found, we have data for determining the longitude of the place.

This is the principle of the method by occultations; the same principle applies to eclipses of the Sun, which may be calculated for any place beforehand, and from the observed time of the occurrence of which, therefore, we may similarly deduce the longitude.

CHAPTER XI.

OF ECLIPSES.

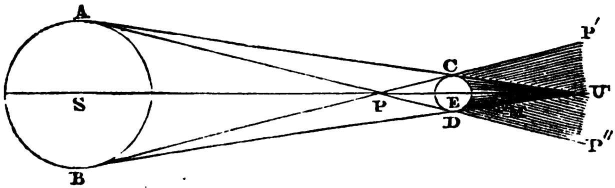
200. *Explanation of the phenomena of a lunar eclipse.*

Eclipses are of two kinds, namely, of the Moon and of the Sun, and may be described in general as being caused, respectively, by the passage of the Moon through the shadow thrown behind the Earth, and by the passage of the Moon between the Earth and the Sun so as to intercept the Sun's light from the Earth. It is manifest that if the motions of the Sun, Earth, and Moon, be accurately known, so that their relative positions at any time can be predicted, it will be only a matter of calculation to determine when an eclipse will take place; but the method of calculation must be very different in the case of a lunar from that of a solar eclipse, and the latter will be very much more complicated than the former, as will be seen when we have described the phenomena more particularly.

LUNAR ECLIPSE.

Let S be the Sun, E the Earth; draw the common tangents to their surfaces ACU, BDU, meeting in U, and the two $APDP''$, $BPCP'$, meeting in P between the Sun and Earth; then the portion of the cone, of which the vertex is U, behind the Earth, is the Earth's shadow, and is called the *umbra;* the portion behind the Earth of the cone, of which the vertex is P, is partially free from the Sun's ray's

in consequence of the intervention of the Earth, and is called the *penumbra.* When the Moon is eclipsed it is



observed to enter the penumbra first, and afterwards the umbra: while in the penumbra the Moon is partly deprived of the Sun's light, and, in consequence, becomes perceptibly less bright; but as soon as it reaches the umbra the portion immersed in the umbra is almost totally obscured, and a small crescent is seen very distinctly separating the small dark portion in the umbra from the comparatively bright part which is in the penumbra; it is, accordingly, not till the Moon has reached the umbra that the eclipse is considered to have really begun. Since the Moon and the Earth's shadow are both moving from west to east on account of the orbital motions of the Moon and Earth, and since the Moon's motion is the more rapid, the Moon enters the shadow on the west of it, and emerges from it on the east; and thus the eclipse of the Moon begins on its eastern limb. On account of the relative magnitude of the Earth's diameter and the distance of the Moon, the distance of the vertex U of the umbra from the Earth is three or four times as great as that of the Moon, and hence the Moon will always pass through the umbra if at the time of geocentric opposition its path is rightly directed.

But there is not necessarily an eclipse of the Moon at each opposition, because the Moon's orbit is inclined to that of the ecliptic, and at the time of opposition it may not be sufficiently near to a *node,* to pass through the umbra.

If the Sun is not too far from the node, the whole Moon will be, at the middle of the eclipse, entirely obscured; for the diameter of the circular section of the umbra at the

Moon's distance varies from about two and a half to three times the diameter of the Moon: the eclipse is then said to be *total*. When the Moon passes through the Earth's shadow, so that throughout the eclipse only a portion of the surface is obscured, the eclipse is said to be *partial*.

201. *Number of eclipses of the Moon possible in a year.*

It is found that, in order that an eclipse of the Moon may be possible, the distance of the Sun from the nearest node of the Moon's orbit, on either side of the node, when the Moon is in opposition to the Sun, must not exceed about $12\frac{1}{2}^{\circ}$; this is called the *lunar ecliptic limit;* the Sun is within this distance from the node, on one side or the other of it, while describing an arc of its orbit equal to twice $12\frac{1}{2}^{\circ}$ or 25°, the time of describing which angle is given by the following proportion:

$$360^{\circ} : 25^{\circ} :: 365{\cdot}24 \text{ days} : \text{the time required};$$

this gives between 25 and 26 days, which is *less* than the time of a synodic revolution of the Moon. If, therefore, a lunar eclipse happens when the Sun is near either node, when the Moon is next in opposition the Sun will be distant from the node by more than $12\frac{1}{2}^{\circ}$; there will, therefore, not be an eclipse of the Moon; and none can happen again at that node, until the Sun has come round to it again.

Thus there can only be one eclipse of the Moon at each node in the course of a year. And there may be none, for the Sun may describe the whole arc of 25° between two successive oppositions of the Moon.

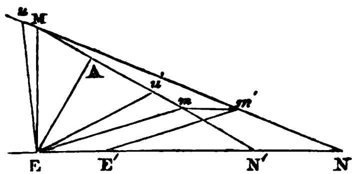
We have here considered the Moon's nodes to be stationary; but they have a *retrograde* motion of about 19° in a year. If, then, an eclipse of the Moon happens at either node at the beginning of the year, the same node will meet the Sun before the end of the year; there may, therefore, be two eclipses at that node in the year. Thus altogether there may be *three* lunar eclipses in a year; and there may be none.

202. *Conditions of possibility of a lunar eclipse. Lunar ecliptic limit.*

We shall endeavour to explain the mode of making the

calculations necessary to determine whether at any given opposition the Moon will be eclipsed, and to what extent.

Let *EN* be a small portion of the path of the centre of the Earth's shadow at the distance of the Moon, considered as a straight line, *MN* a portion of the path of the Moon's



centre, *M* being the position at the time of opposition, and *N* the node. Then it will be convenient to suppose the Earth to remain fixed, and the Moon to move in an imaginary orbit *MN′*, called a *relative* orbit, such that the distance between the centres of the Moon and shadow shall always be the same as they actually are. Let *m′* be the Moon's centre when the centre of the shadow is at *E′*; join *E′m′*, take *m′m* equal and parallel to *EE′*, and join *Em*; then *Em* will be equal and parallel to *E′m′*, and *m* will therefore be a point in the relative orbit. It is not difficult to see that the relative orbit *MN′* will be a straight line, and its inclination *MN′E* to the plane of the ecliptic may be calculated from a knowledge of the inclination of the Moon's orbit and of the relative velocities of the Earth and Moon.

The radius of the umbra at the Moon's distance, or rather the angle subtended by that radius at the Earth, may be easily calculated from the parallaxes of the Sun and Moon and their apparent diameters; all which quantities are known and registered.

Now draw *EA* perpendicular to the relative orbit; then since *ME*, which is the Moon's latitude at the time of opposition, and *EMA* are known, *EA* may be calculated. *EA* is the nearest approach of the centres of the Moon and the umbra; if then *EA* be greater than the sum of the radii of the Moon and the umbra, there will be no eclipse, if less there will be an eclipse of greater or smaller degree of obscuration according to the value of EA. If we draw two

lines Eu, Eu', each equal to the sum of the radii of the Moon and umbra, then u and u' will be the positions of the Moon's centre at the commencement and termination of the eclipse respectively; and by calculating these positions we can easily determine the times of the commencement and termination.

The value of EN—the distance between the centres of the Earth's shadow and the node of the Moon's orbit at opposition—when EA is equal to the sum of the radii of the umbra and the Moon, is its greatest value consistent with the possibility of a lunar eclipse; this value is called the *lunar ecliptic limit.*

203. *Places at which a given lunar eclipse is visible.*

Since a lunar eclipse is caused by an actual deprivation of the Sun's light, in order to determine the places at which a given lunar eclipse will be visible, it is only necessary to determine the places which will have the Moon above their horizon at the time. The calculation is easily made, but for practical purposes it is sufficient to proceed as follows: Take a common terrestrial globe, determine upon it the Moon's place at the commencement of the eclipse, then all places on the hemisphere lying round this point will see the commencement of the eclipse; in like manner determine the hemisphere from all places of which the termination is visible; then the whole of the eclipse will be visible from all places which are common to these two hemispheres.

204. *Greatest possible duration of the total obscuration of the Moon in a total lunar eclipse.*

The duration of the totality of a total eclipse of the Moon will vary with the distance EA of the relative orbit of the Moon's centre from the centre of the shadow. If this distance be greater than the difference between the radius of the shadow and of the Moon there is no total eclipse; if less, there is a total eclipse; if the relative orbit pass through the centre of the shadow the duration of the total eclipse is the time between the first and the last *internal* contact, and is therefore the time which the *Moon's centre* takes to describe a space equal to twice the

difference between the radii of the Earth's shadow and the Moon; this, which is the greatest duration possible, is found to be rather over two hours.

SOLAR ECLIPSE.

205. *Number of solar eclipses in a year.*

The calculation of a solar eclipse is more difficult than that of a lunar, because the Moon by its interposition between the Earth and the Sun will intercept the Sun's light from some portions of the Earth's surface, but not from others, and the Sun may be visible at a given place during an eclipse, although the eclipse may not be visible.

In order that a solar eclipse may be possible, the distance of the Sun from the nearest node of the Moon's orbit, at the time of conjunction of the Moon, must not exceed about $18\frac{1}{2}^\circ$; this is the *solar ecliptic limit.* It is hence found, in the same manner as for lunar eclipses, that the Sun takes *more* than the time of a synodic revolution of the Moon to describe this distance on both sides of a node. Hence, taking account of the regression of the nodes, there may be three solar eclipses, and *must* be one, at the same node, in a year; thus at the two nodes there may be five solar eclipses, and must be two.

206. *Total, annular, and partial eclipses.*

On account of the excentricities of the orbits of the Moon and the Earth, the ratio of the distances of the Sun and Moon from the Earth, at conjunction, will vary slightly, so that the apparent diameter of the Moon will be sometimes greater, and sometimes less, than that of the Sun. When there is an eclipse of the Sun, the exterior cone circumscribing the Sun and Moon meets the Earth in a curve, at any point on which the discs of the Sun and Moon will be seen to touch each other internally; and at any point within the area enclosed by the curve the Moon will completely hide the Sun, or will leave a ring uneclipsed, according as the angular diameter of the Moon is greater or less than that of the Sun. In the former case the eclipse is *total,* and in the latter it is said to be *annular.* At points beyond this area, and within a short distance of it,

the eclipse is *partial*, a portion only of the Sun's disc being hidden. The curve is in motion throughout the eclipse, and the whole area enclosed by it in all portions will contain the places at which the eclipse is total or annular.

207. *Greatest possible duration of the total obscuration of the Sun in a solar eclipse.*

Since the Moon moves from west to east more rapidly than the Sun it gains on the Sun, and an eclipse of the Sun begins therefore when the Moon's east limb begins to cover the west limb of the Sun. The duration of the totality of a total eclipse is greatest when the line of nodes of the Moon's orbit at the time passes through the Sun's centre, and the apparent diameter of the Moon is its greatest possible, and that of the Sun its least; and the duration of the totality is the time taken by the Moon to gain on the Sun by a space equal to the difference between these apparent diameters. Now, the difference between the greatest angular diameter of the Moon and the least of the Sun is about two minutes of space; and in one lunation, or $29\frac{1}{2}$ days, the Moon moves through 360^{0} relatively to the Sun; hence, it will be found that the Moon takes about four minutes of time to move relatively to the Sun through two minutes of space. Thus the totality of a total solar eclipse cannot last longer than four minutes.

208. *Circumstances of a solar eclipse at different places.*

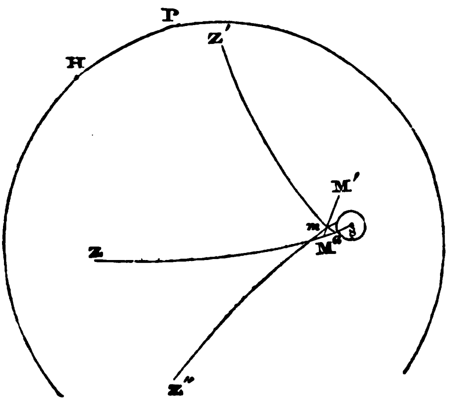
We shall attempt to give some account of the mode of calculating the circumstances of a solar eclipse, premising that as we have already supposed the Earth to remain fixed and the Moon to move in a relative orbit, so here we shall suppose the Sun to be fixed and the Moon to be apparently depressed by the difference of the solar and lunar parallax, or (as we may call it) the *relative* parallax. This relative parallax, as well as the apparent diameters of the Sun and Moon, are known from the Nautical Almanac or some equivalent work.

Let P be the pole of the equator, and for distinctness' sake, let the plane of the paper be the plane of the *solstitial* colure: S the Sun's centre, supposed fixed, MM'

the Moon's relative orbit; round S draw a small circle, having for its radius the sum of the apparent semidiameters of the Sun and Moon; then to any place, which is so situated that the Moon's centre can be sufficiently depressed by parallax to make it fall within this small circle, the eclipse will be visible.

Let us find the place at which the eclipse will be first seen. Draw SaM an arc of a great circle, and make Ma equal to the relative parallax, and produce SM to Z, so that $Za = 90^0$, then Z will be the zenith of the place required, for to such a place M will be depressed towards S by the whole relative horizontal parallax.

Again, suppose we wish to determine those places on the Earth's surface at which the first contact for different positions of the Moon first becomes visible. Let m be any position of the Moon in the relative orbit; then from m we can draw, in general, two arcs of great circles each equal



to the relative horizontal parallax to meet the small circle round S, and if we produce these to Z' and Z'', making $mZ' = mZ'' = 90^0 -$ the relative parallax, the Moon will appear depressed as much as possible to these two places, and *therefore* for the position m of the Moon Z', Z'' are

the zeniths of the two places to which an apparent contact will be first visible. And so we may trace a curve on the Earth's surface including all places determined by this construction.

In like manner any other problem may be solved; the most general perhaps is this, To find all places on the Earth's surface at which a given portion of the Sun's surface will appear obscured at a given time.

By methods founded upon the principles which we have endeavoured briefly to describe, maps are constructed, such as those in the Nautical Almanac, exhibiting curves on the Earth's surface comprehending all places for which an eclipse will be visible in a given degree.

209. *Saros. Metonic Cycle.*

It is found that 19 synodic revolutions of the Moon's nodes, or 19 of the intervals which elapse between successive conjunctions of the node and Sun, are within a fraction of a day the same as 223 synodic revolutions of the Moon; each being between 6585 and 6586 days. At the end of this period therefore the Moon, Sun, and Moon's nodes all return to almost precisely the same positions with respect to one another as they had at the beginning; hence, nearly the whole series of eclipses which happened during that interval will recur in the same order. This cycle was known to the Chaldeans, and was called the *Saros.*

Another cycle, known to the ancients, is one of 19 Julian years, or $19 \times 365\frac{1}{4}$ days, which differs by only about an hour and a half from 235 synodic revolutions of the Moon. At the end of this period, therefore, both the Moon and Sun are in almost precisely the same parts of the heavens as at the beginning; the new and full Moons consequently begin to recur on the same days of the month and year.

This period is called the *Metonic cycle.*

EXAMPLES.

1. Assuming the Earth's radius to be 3980 miles, shew that to an eye raised n feet above the level of the sea, an object placed at that level, and at about $\sqrt{\frac{3n}{2}}$ miles from the eye, will just appear on the horizon.

2. If in latitude 45^0, the transit of a star in the equator be unaffected by the combined effects of the errors of level and deviation, prove that these errors will be very nearly equal to each other.

3. How might the inclination of the ecliptic and equator to one another be ascertained by observations made at the times of the solstices? If the intersection of the plane of the ecliptic with the heavens were visible in the sky, of what sort would be its apparent daily motion?

4. When the meridian altitude of a heavenly body is known, what other elements are required in order to determine its position among the stars?

5. At a place on the equator the lengths of the shadows at noon of a vertical rod are h and h', towards the north and south respectively, on consecutive days; determine approximately the time of the vernal equinox.

6. Find the latitude of a place in which the longest day contains 16 hours.

7. How is it inferred that the axis of the Earth's rotation always coincides sensibly with the same line of particles in the Earth, and that it does not rapidly change its direction in space?

8. When the Sun has a given N.P.D. shew at what places on the Earth it is visible during (1) 24 hours, (2) 12 hours continuously.

9. Find the declination of the Sun when for a given place within the Arctic Circle the Sun at mid-day just appears above the horizon.

10. Orion's belt being in the equator and having R.A. 5h. 30m., during what part of the night will it be visible at the vernal and autumnal equinoxes?

11. If a star whose R.A. is $19^0\ 25'$ pass over the meridian 2 h. 18m. of sidereal time before the Sun, what is the Sun's R.A. when on the Meridian?

12. The constellation 'ursa major' is visible at night all the year round; 'piscis australis' only during the summer; and 'orion' during the winter; in what parts of the heavens are these constellations situated respectively?

13. Within what limits of latitude will the ecliptic be perpendicular to the horizon twice in a day?

14. What is the latitude of a place with the horizon of which the ecliptic can coincide? At what time of the day does it happen for such a place?

15. At a place on the Arctic circle at what point will the Sun rise immediately before the summer solstice?

16. If a star whose R.A. is $19^0\ 25'$ pass over the Meridian 2 h. 18 m. of sidereal time before the Sun, what is the Sun's R.A. when on the Meridian?

17. Shew how to determine the position of the ecliptic at a given time of the year and hour of day. Shew in a figure the position of the ecliptic at 3 P.M. on June 5 for a place in latitude 52^0 N.

18. Shew how the phenomena of the seasons would be altered if the Earth's axis were inclined to the plane of the ecliptic at an angle of 90^0, or of 45^0, or of 0^0, the axis being in each case supposed to remain parallel to itself.

19. If the angle between the equator and the ecliptic were 15^0, what fractional part of the Earth's surface would be included in the torrid zone, the temperate zone, and the frigid zone respectively?

20. If at midnight, at the time of the summer solstice, a meteor, moving from the south to the north, perpendicular to the Earth's orbit, with the Earth's velocity, pass through the zenith, shew in what direction it will appear to move.

21. In 365 d. 5 h. 48 m. the Sun's longitude is increased by 360^0; what is his mean daily motion?

22. Account for the fact that the time of the Sun's setting as given in the ordinary Almanacs is not the latest on the longest day.

23. The mean time being 8 hours, find the corresponding sidereal time, having given the Sun's mean daily motion in R.A. $59'\ 8{\cdot}33''$, and the mean R.A. at the previous mean noon $185^0\ 45'$.

24. The Sun rose one morning at 8 h. 7 m., and set the same evening at 4 h. 5 m. What was the value of the equation of time on that day?

25. Assuming the length of a sidereal year to be 365 d. 6 h. 9 m. 10·7 s. in mean solar time, find approximately the difference between a sidereal and a mean solar day.

26. Assuming the actual value of the tropical year to be 365·24224 days, prove that the error of the Gregorian correction, had it been adopted at the commencement of the Christian era, would amount to 13 days in the year 50,000.

27. On June 8, 1839, the Sun rose at 3 h. 46 m., and set at 8 h. 12 m.; what was then the value of the equation of time?

28. Will the greatest difference between the clock and the Sun be when the clock is before or behind the Sun?

29. The equation of time at noon on one day is 3 m. 14 s., and at the succeeding noon is 3 m. 12 s.; what time ought a correct watch to shew when a sun-dial marks 6 o'clock on the evening of the former day?

30. At 2 P.M. by an ordinary clock, a sidereal clock indicated 20 h.: about what time of the year was it?

31. What is the time at a place 30^0 west longitude, when it is mid-day at a place 90^0 east longitude?

32. The difference between the parallaxes of the Sun and Mercury at a transit of Mercury was observed to be 4″, subject to an error not greater than ·01″, and the calculated value of the Sun's horizontal parallax was 8·65″; shew that this quantity is correct within ·022″. Had the transit been that of Venus, the observed difference of the parallaxes 21·5″, and the calculated parallax of the Sun correct within the same limits as before, what would have been the greatest error to which this quantity might have been subject?

33. What is the cause of twilight; and why is its duration much less in the tropics than in the higher latitudes?

34. Assuming that light passes from the Sun to the Earth in 8 m. 13 s., that the Moon's period is $27\frac{1}{3}$ days, and her distance from the Earth a 400th part of the Sun's distance; find the mean amount of the Moon's aberration.

35. Shew that tables which serve to give the effects of aberration on the position of a star will also serve to give the effects of annual parallax three months previously.

36. Prove that the ratio of the axis of the aberration-ellipse, of a star whose latitude is λ, is $1 : \sin\lambda$.

37. At what seasons of the year is the aberration of a star in the position of the first point of Aries greatest?

38. A star situated in the solstitial colure passes the meridian at 6 A.M. Shew that its R.A. is not affected by aberration. Is its declination increased or diminished by it?

39. Under what circumstances is a planet's apparent place not changed by aberration?

40. Shew that at any given time all stars which lie in a certain great circle have no aberration in R.A. Also give a geometrical construction for finding stars which have no aberration in N.P.D. at a given time; and prove that the locus of such stars becomes a great circle twice a year.

41. Supposing the mass of the Sun to have been greater than it now is, and the Earth to have described the same orbit which it now describes, in what respect would the aberration-curve of any given star have been different from what it now is?

42. Shew that, if the annual parallax be taken into account, the aberration-curve of a star cannot be very different from what it would be if the annual parallax were neglected.

43. If two planets describe circles in one plane, when will the aberration in the position of one as seen from the other be greatest and least?

44. How will the position of a star in the plane of the ecliptic as it appears to a spectator on the Earth be affected by aberration, as the difference of R.A. of the star and the Sun increases from 0 h. to 24 h.?

45. Find the least diurnal rotation of the Earth, which will render sensible to an observer at the equator the diurnal aberration, the least appreciable angle being $1''$.

46. In what positions of a star are its R.A. and N.P.D. respectively unaffected by annual parallax?

47. Give the reasons for the slight difference in the sidereal times of transit of a star as taken at different times of the year, supposing the instrumental and clock-errors to have been corrected.

48. What must be the relation between the distances from the Sun of a superior and inferior planet, that their synodic revolutions may be equal?

49. Having given that the mean motions of the Earth and Venus about the Sun are as 8 : 13; shew that Venus will be at her greatest angular distance from the Sun at sunset, at intervals of 584 days nearly.

50. If Venus is a morning star and stationary, will it begin to move forwards or backwards among the stars?

51. If Jupiter revolves round the Sun in 4320 of our days and round his axis in 10 hours; find by how much his mean solar exceeds his sidereal day.

52. The distance of Venus from the Sun being assumed to be ·723 of the Earth's, find how long after conjunction she will be stationary.

53. Shew that the period of the retrograde motion of an inferior planet is to that of its direct motion as $\pi - 2\alpha$ is to $\pi + 2\alpha$, where α is its extreme elongation; the orbits of the *Earth* and planet being supposed circular.

54. The parallaxes of the Sun and Moon being respectively $8 \cdot 86''$ and $57'$, find approximately the ratio of the distances of the Sun and Moon from the Earth, and thence shew that the Moon's orbit in space is always concave to the Sun.

55. Shew that the full Moon in winter is longer above the horizon than in summer.

56. At the Moon's rising, shortly after new Moon, will the horns of the crescent point upwards or downwards?

57. Why is the angular magnitude of the Moon less when on the horizon than when at a great altitude?

58. Shew how the existence of the Moon's libration in longitude would be recognized by an observer of the Earth from the Moon.

59. What must be the approximate age of the Moon that she may be seen in the south at seven o'clock in the morning at the time of the vernal equinox? Will the convexity of the crescent appear to a spectator on his right hand or on his left?

60. Explain the variations of the times and places of setting of the Harvest Moon.

61. The angular distance of Aldebaran from the Moon's centre was observed at a certain place at 3 h. 40 m. to be $60^\circ\ 14'$; at Greenwich, at noon and at 3 h., the distances of the same objects were $65^\circ\ 9'\ 30''$ and $66^\circ\ 41'\ 30''$ respectively; determine the longitude of the place.

62. Supposing the Moon to move in the ecliptic, determine approximately the duration of a solar eclipse; taking the angular diameter of the Sun and Moon to be $30'$ each, and the radius of the Moon to be $\frac{3}{11}$ that of the Earth.

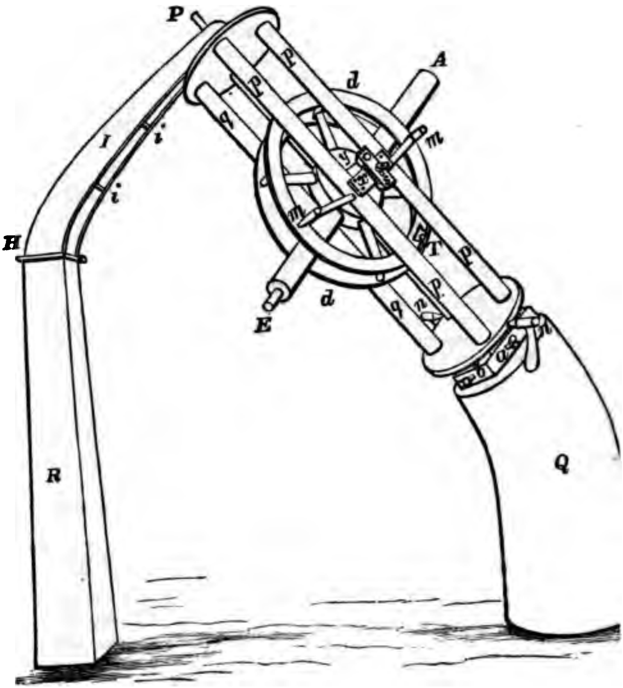
63. In a solar eclipse, shew that the shadow moves eastwards on the Earth's surface.

64. Shew that the ecliptic limits are greater for a solar than for a lunar eclipse.

65. Two stars S, S', which are very near each other, are observed to have the same azimuth. When S, after passing the meridian, has again the same altitude as before, S' has also that altitude. Prove that the ratio of the differences of the R.A.s and N.P.D.s of S and S' is nearly $1 : \cos\delta$, where δ is the declination of S; and that, if the times of the observations be nearly those of the rising and setting of S', and l be the latitude of the place of observation,

$$\sqrt{2}\sin l = \cos\delta, \text{ nearly.}$$

66. What knowledge of the motions of the heavenly bodies can we obtain without knowing the form and magnitude of the Earth? What additional information is deduced from knowing these circumstances?



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